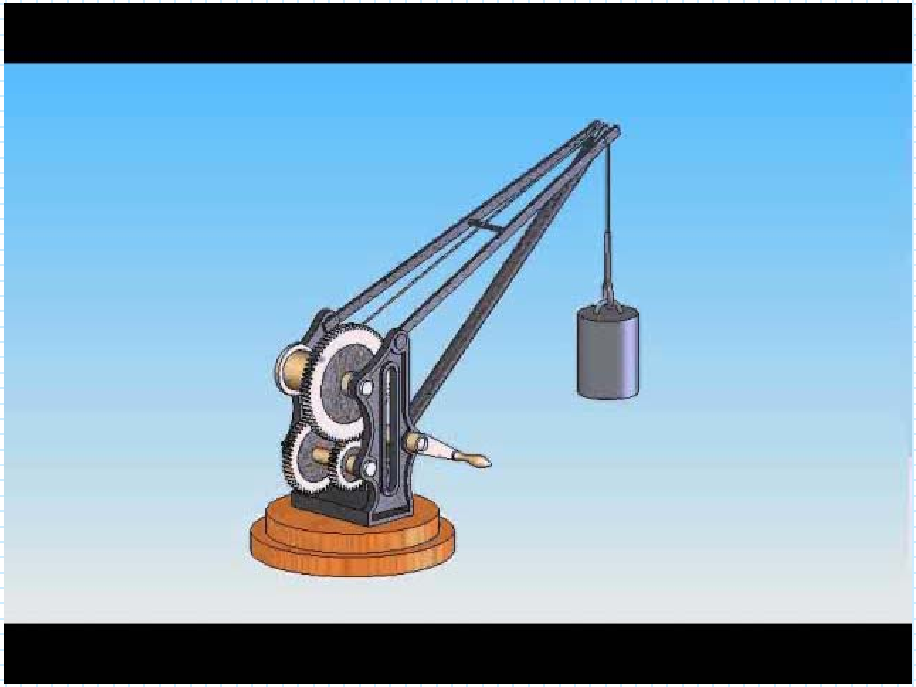
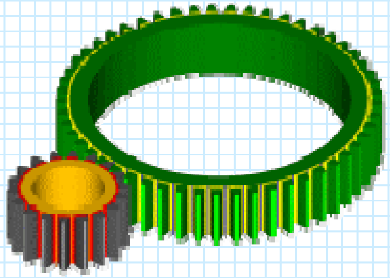
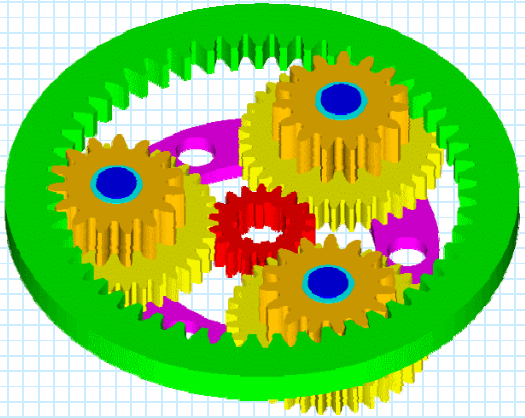
Introduction to Gear Trains

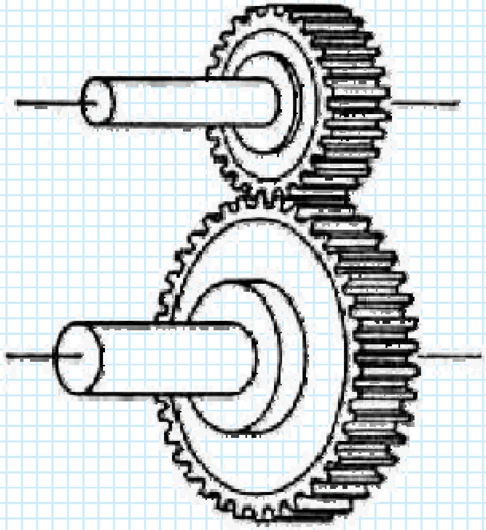




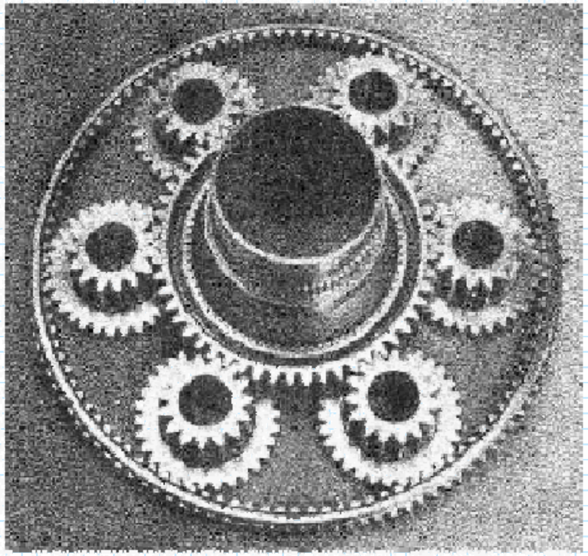


Most of the pictures and animations are from Norton, "Design of Machinery" McGrawHill, 2004

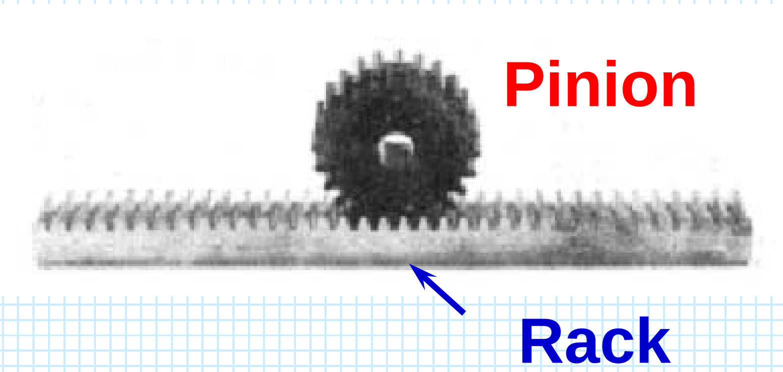
GEAR TRAINS



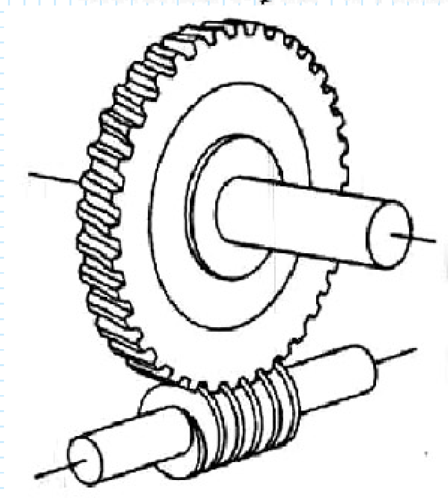
External Spur Gears



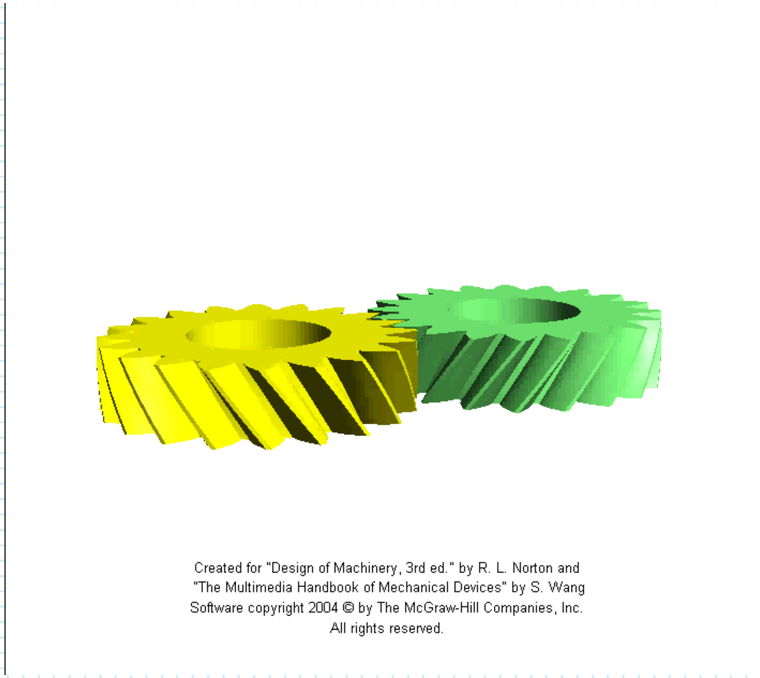
Internal Spur Gears

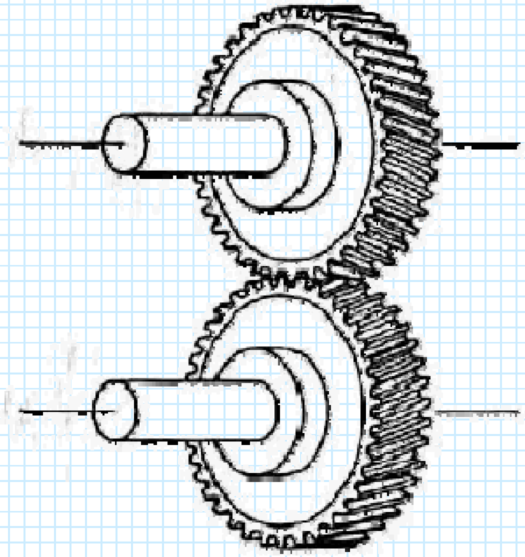


Rack & Pinion



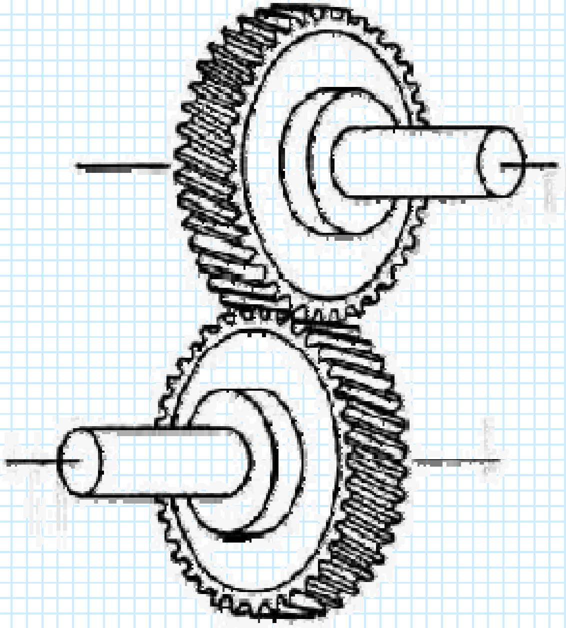
Worm Gear



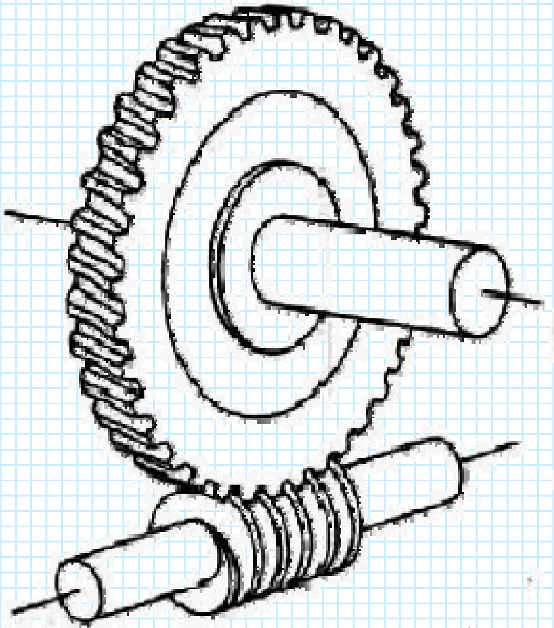


Helical (Parallel Shaft)

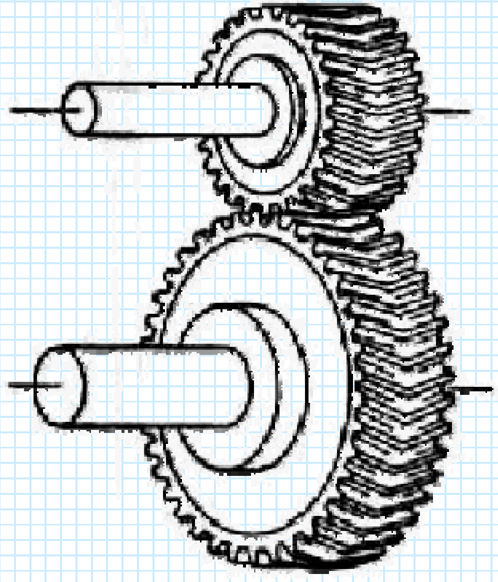
quieter than
spur gear



Helical (Crossed Shaft)

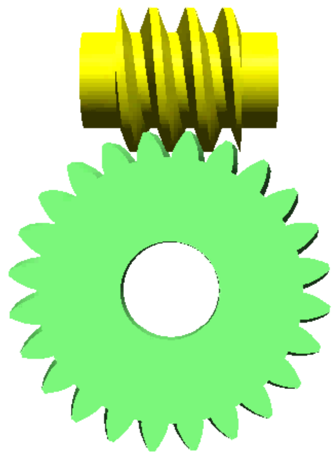


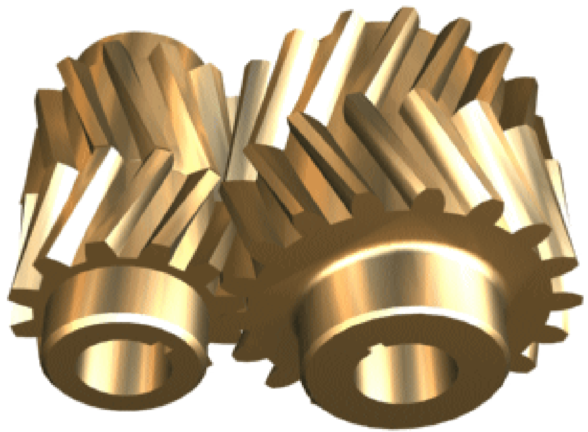
Worm and Gear

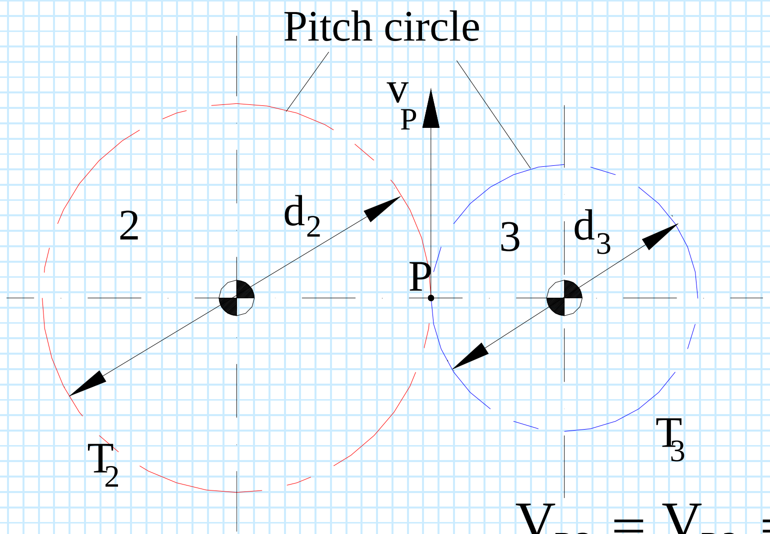


Herringbone Gears

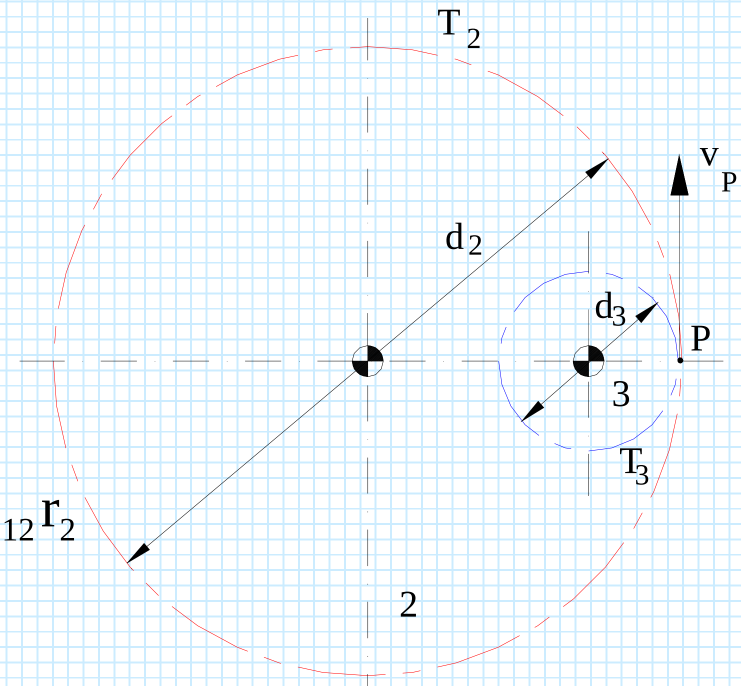
Very high gear ratio is possible in small package. Allow one directional drive: worm $\rightarrow$ worm wheel







$$V_{P3} = V_{P2} = \omega_{13} r_3 = \omega_{12} r_2$$



$$R_{23} = \frac{\omega_{13}}{\omega_{12}} = \frac{n_{13}}{n_{12}} = -\frac{r_2}{r_3} = -\frac{d_2}{d_3}$$

$$R_{23} = \frac{\omega_{13}}{\omega_{12}} = \frac{n_{13}}{n_{12}} = +\frac{r_2}{r_3} = +\frac{d_2}{d_3}$$

Two gears can only mesh if they have the same "circular pitch", "diametral pitch" or "module"

Circular pitch, c_p: $2\pi r = c_p T$ (T= no of tooth on the circumference)

c_p= ***the distance between a point on one gear tooth and the same point on the next gear tooth (c_p is measured in inches).***

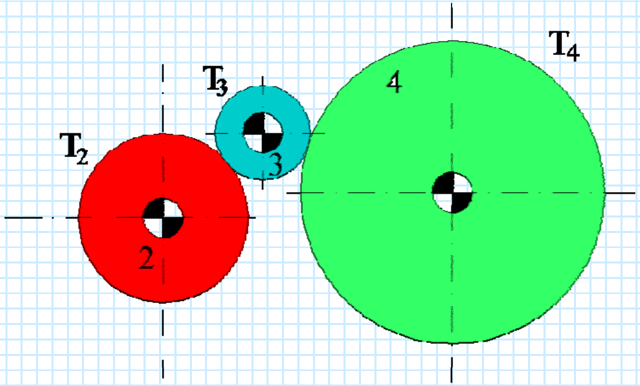
Diametral Pitch P_D= T/D

Module, m: $\pi D = m\,T$ (***m is measured in mm***)

The + sign is used here to take into account the direction of rotation.

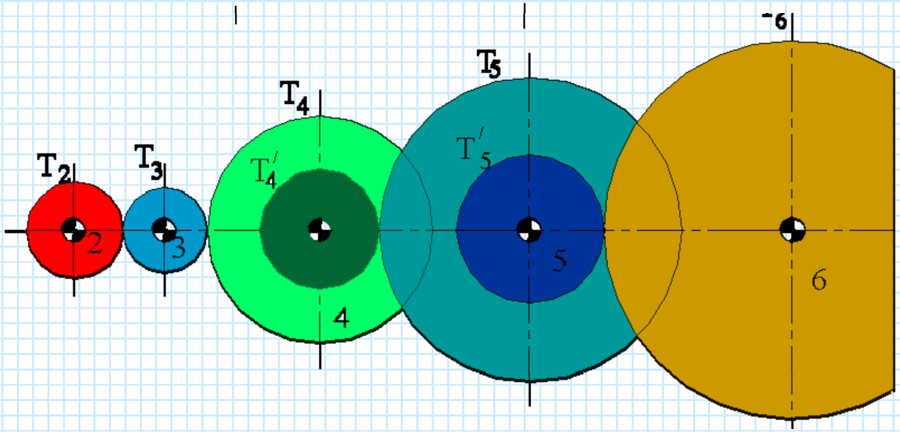
Hence: $$R_{23} = \frac{\omega_{13}}{\omega_{12}} = \frac{n_{13}}{n_{12}} = \pm\frac{r_2}{r_3} = \pm\frac{d_2}{d_3} = \pm\frac{T_2}{T_3}$$

Simple Gear Trains



$$R_{24} = R_{23}R_{34} = \left(-\frac{T_2}{T_3}\right)\left(-\frac{T_3}{T_4}\right) = \frac{T_2}{T_4}$$

Gear 3 is "idler". Used to change the direction of rotation or to connect two shafts at a distance.



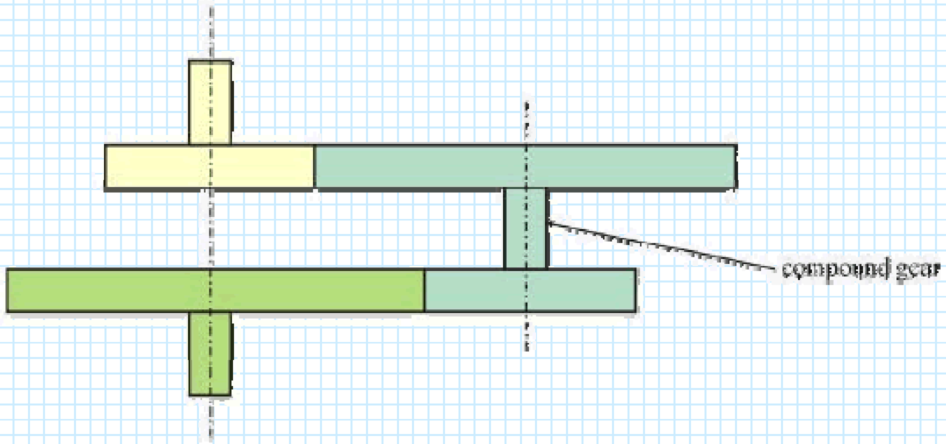
Simple "Compound"Gear Train

$$R_{26} = \frac{n_{16}n_{15}n_{14}n_{13}}{n_{15}n_{14}n_{13}n_{12}} = R_{23}R_{34}R_{45}R_{56} = (-1)^4 \frac{T_2T_3T_4'T_5'}{T_3T_4T_5T_6}$$

In general:

$$R_{ij} = \frac{n_{1j}}{n_{1i}} = (-1)^k \frac{\text{Product of driving gear tooth numbers}}{\text{Product of driven gear tooth numbers}}$$

k = number of external gear meshes.

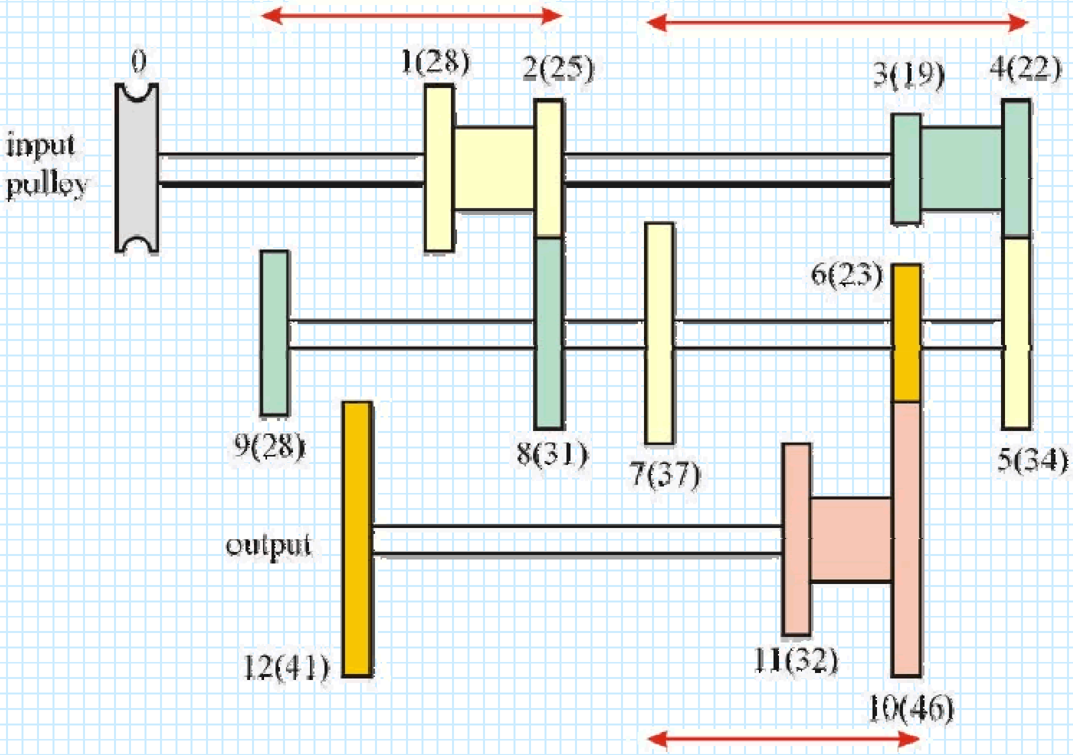


Reverted Gear

Used in automotive transmission:

- compact, save space

Revert = go back to a previous state

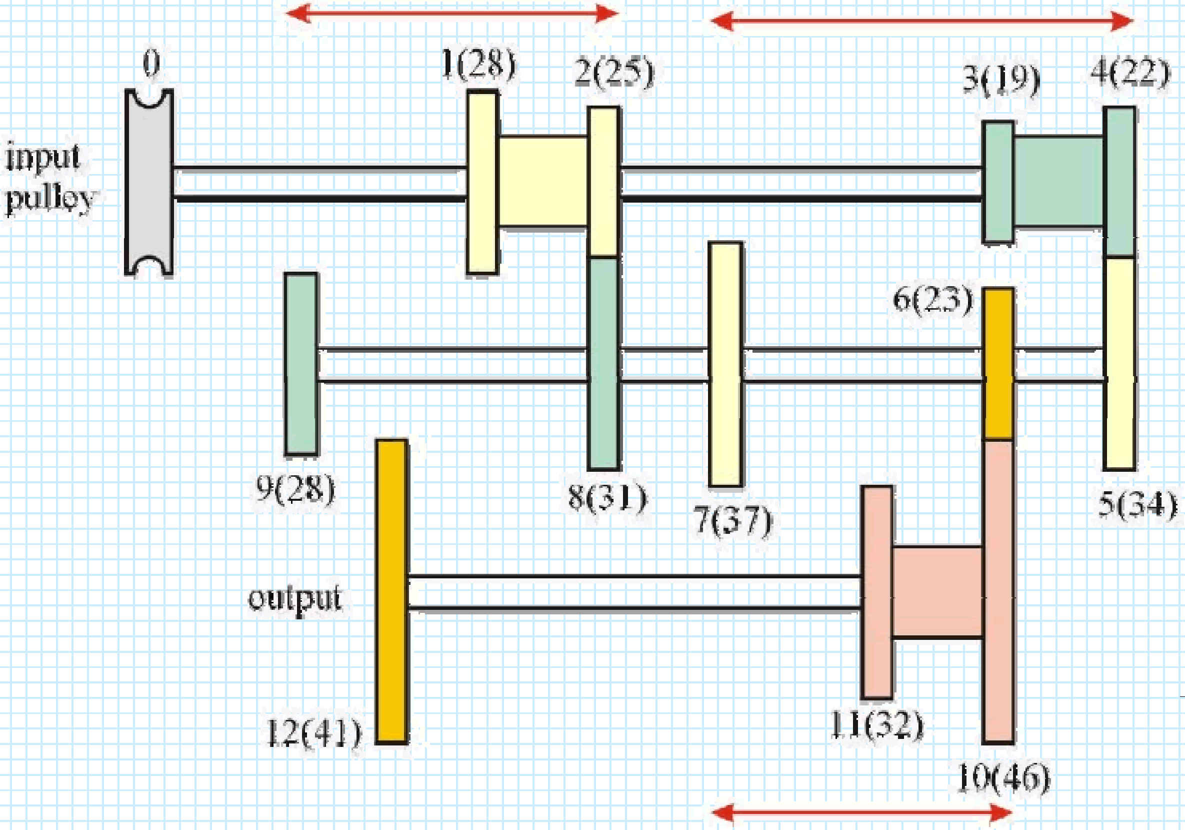


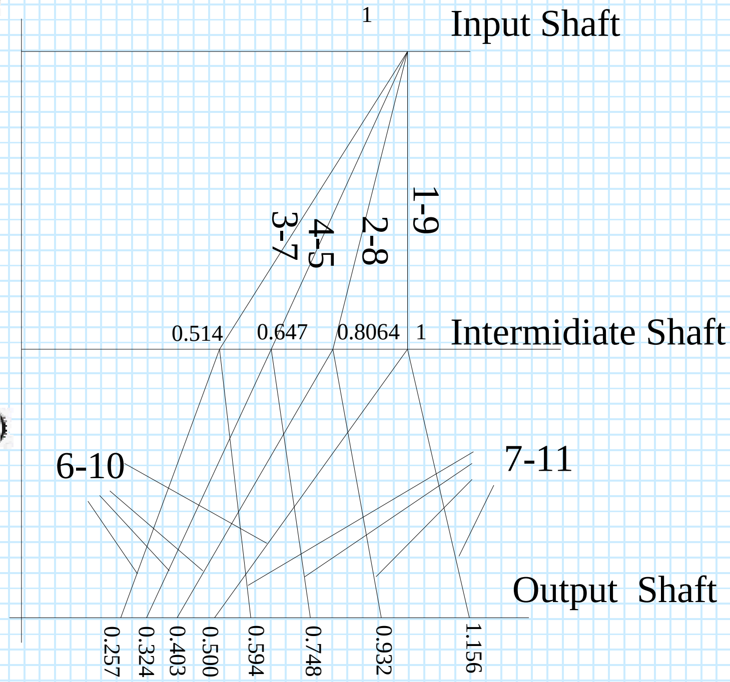
Speed change gear box

0-4-5-6-10-12

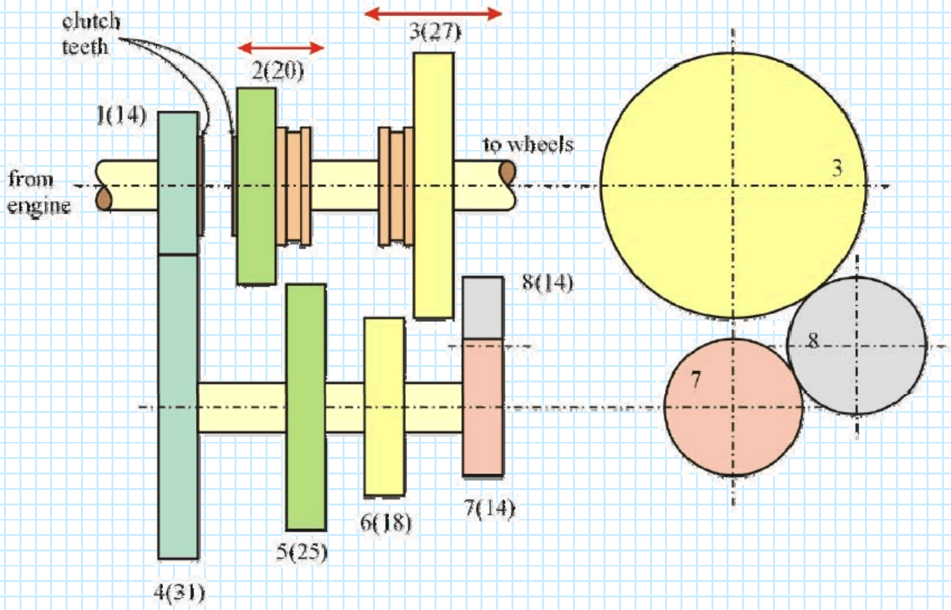
$$\frac{\omega_{output}}{\omega_{input}} = (-1)^2 \frac{22 \bullet 23}{34 \bullet 46} = \frac{11}{34} = 0.323$$

Important: Gears are numbered. Not the links!!


0
1(28)
2(25)
3(19)
4(22)
input pulley
6(23)
9(28)
8(31)
7(37)
5(34)
output
12(41)
11(32)
10(46)


1
Input Shaft
1-9
2-8
4-5
3-7
0.514
0.647
0.8064
1
Intermidiate Shaft
6-10
7-11
Output Shaft
0.257
0.324
0.403
0.500
0.594
0.748
0.932
1.156

Automotive Transmission



http://auto.howstuffworks.com/transmission.htm

Low gear: Gear 3 meshes with gear 6, power flows 1-4-6-3

$$\frac{\omega_{out}}{\omega_{in}} = \frac{14}{31} \bullet \frac{18}{27} = 0.301$$

Second Gear: gear 2 meshes with gear 5, power flows 1-4-5-2

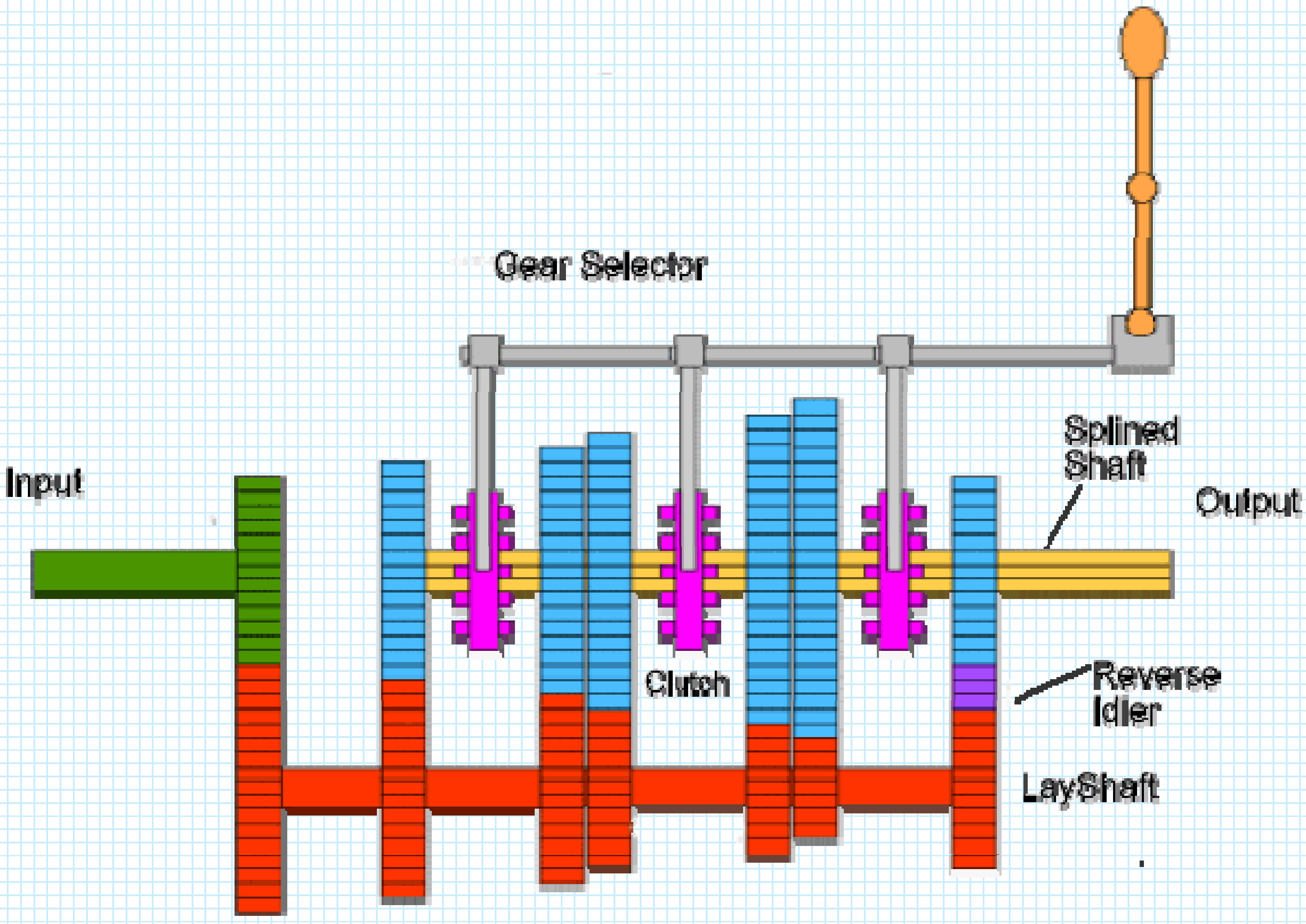
$$\frac{\omega_{out}}{\omega_{in}} = \frac{14}{31} \bullet \frac{25}{20} = 0.564$$

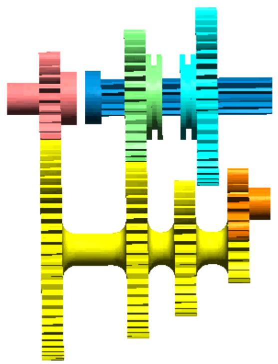
High Gear: gear 2 is shifted so that the clutch teeth on the end of gear 2 mesh with the clutch on gear 1 (direct drive)

$$\frac{\omega_{out}}{\omega_{in}} = 1$$

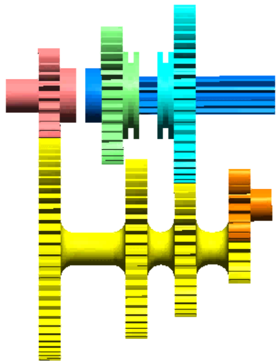
Reverse gear: gear 3 is shifted to mesh with gear 8, power flows 1-4-7-8-3.

$$\frac{\omega_{out}}{\omega_{in}} = -\frac{14}{31} \bullet \frac{14}{27} = 0.234$$


Gear Selector
Input
Splined
Shaft
Output
Clutch
Reverse
Idler
LayShaft



Created for "Design of Machinery, 3rd ed." by R. L. Norton and "The Multimedia Handbook of Mechanical Devices" by S. Wang



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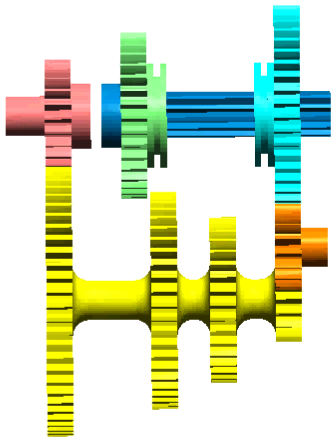

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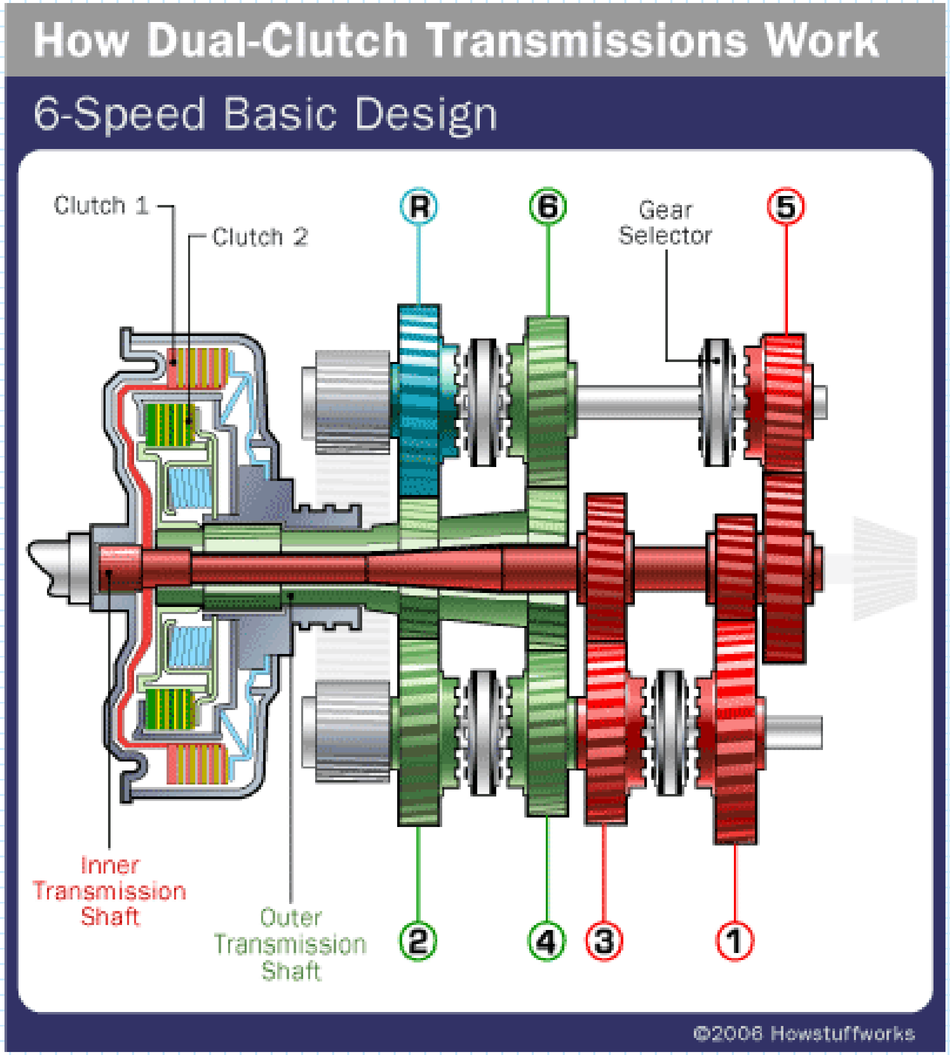
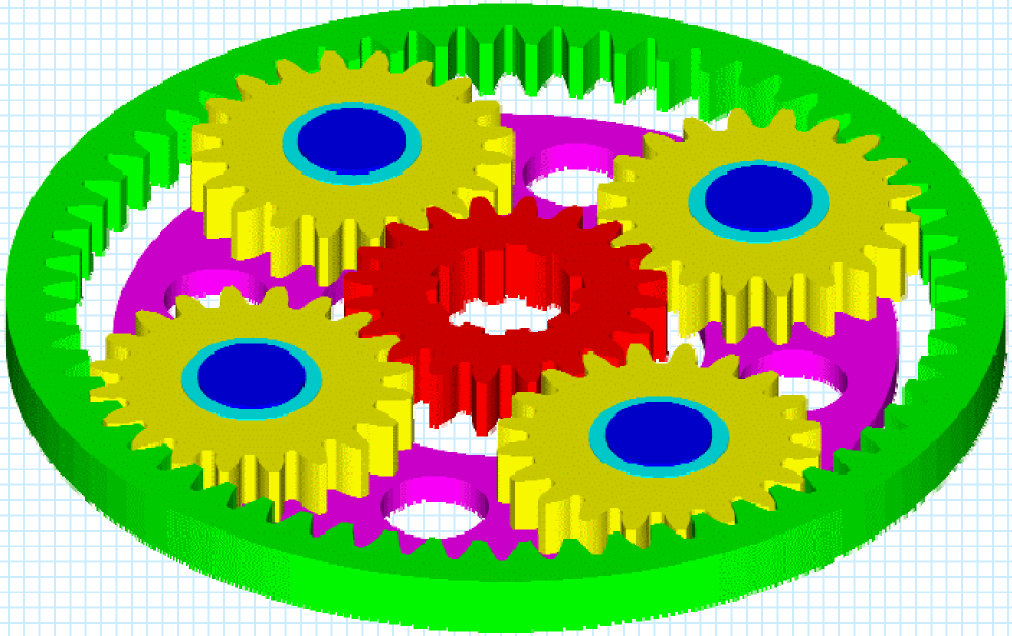
Dual Clutch Transmission



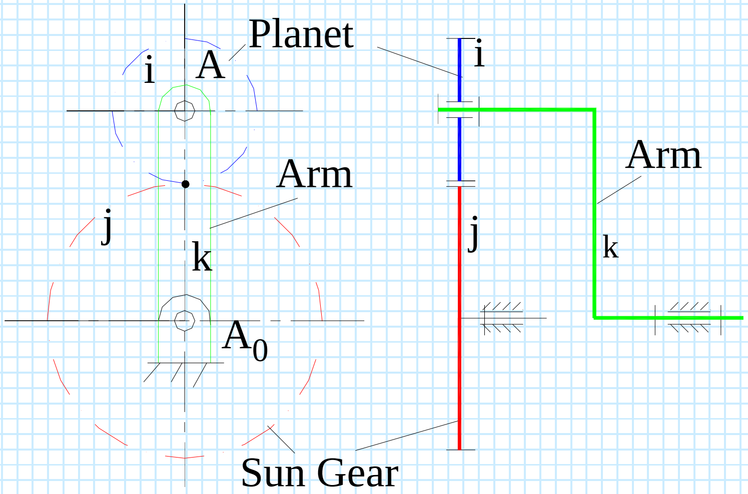
From: How stuffs work: **http://auto.howstuffworks.com/dual-clutch-transmission1.htm**

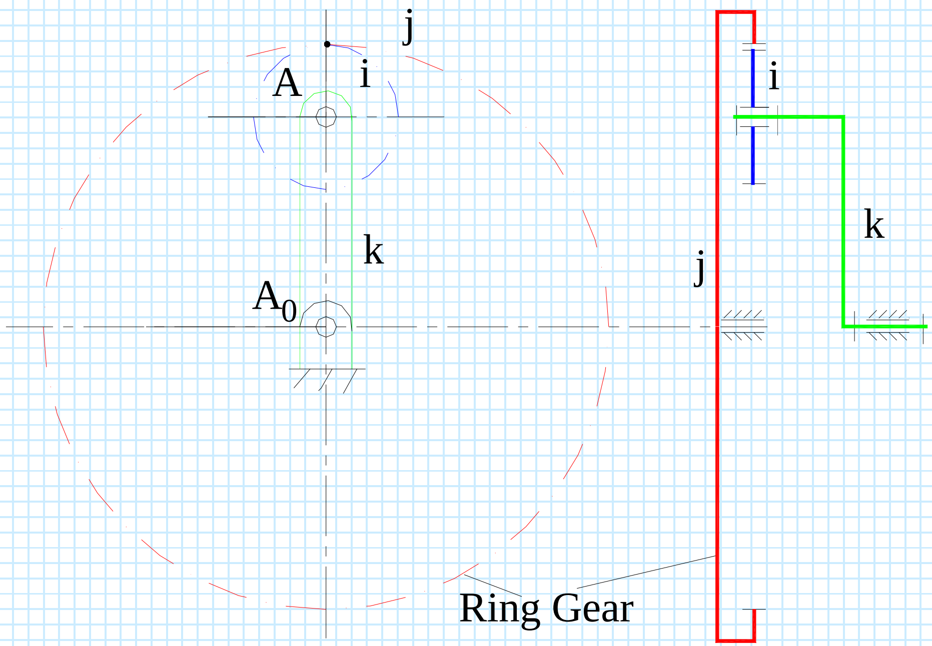
PLANETARY GEAR TRAINS

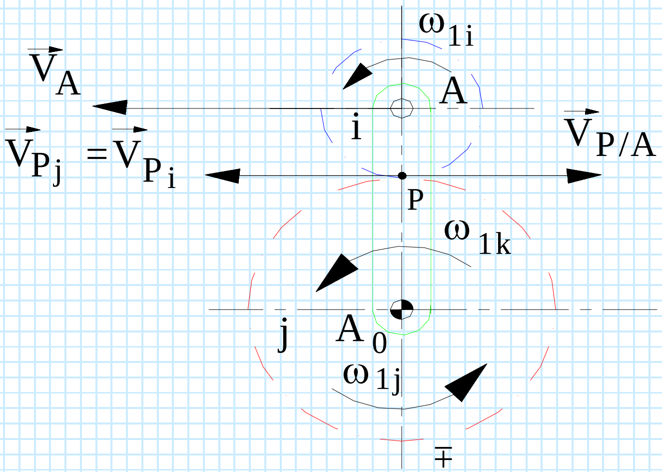
The axis of one of the gear (planet) is not fixed

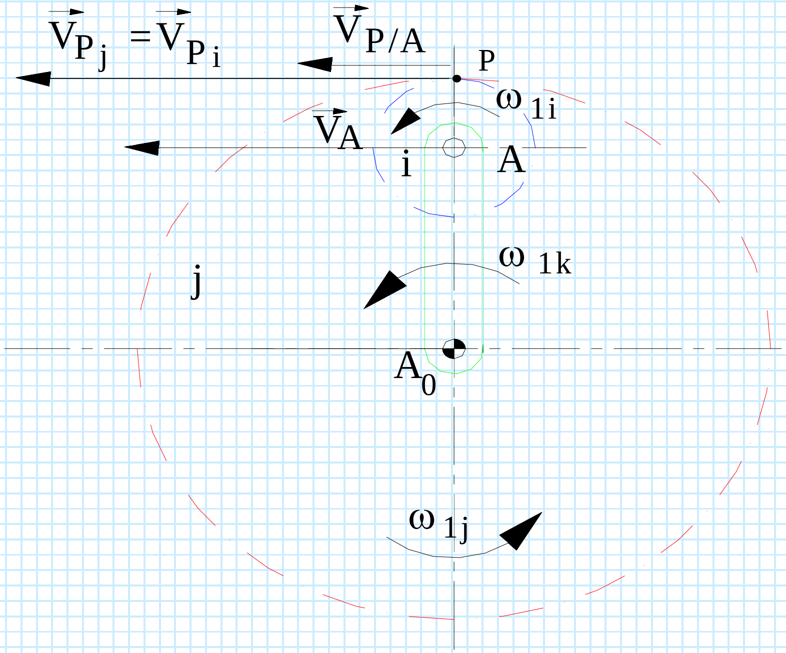


Planetary Gear Trains









$$\mathbf{V}_{Pi} = \mathbf{V}_{Pj} = \mathbf{V}_{A} + \mathbf{V}_{P/A}$$

$$v_{Pj} = v_{Pi} = \omega_{1j} r_j$$

$$v_A = \omega_{1k}(r_j \mp r_i)$$

(- if external, + if internal mesh)

$$v_{P/A} = \pm\omega_{1i} r_i$$

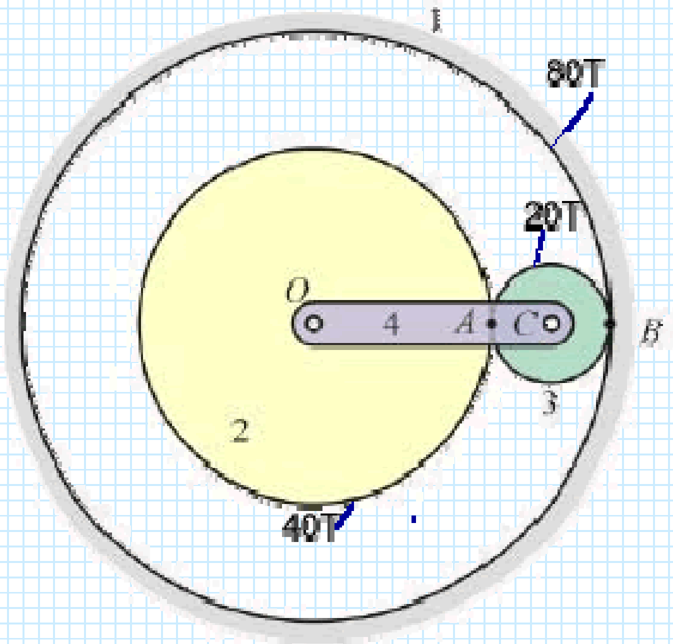
$$\omega_{1j} r_j = \omega_{1k}(r_j \mp r_i) \pm \omega_{1i} r_i$$

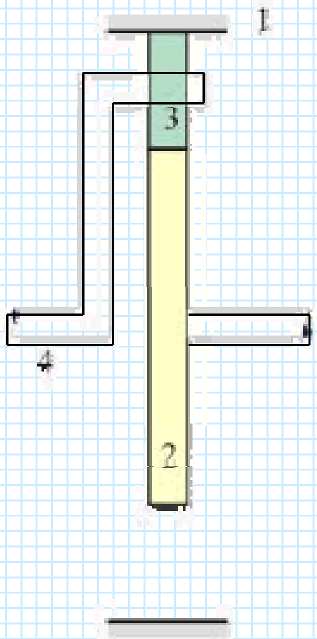
$$\pm\frac{r_i}{r_j} = \pm\frac{d_i}{d_j} = \pm\frac{T_i}{T_j} = R_{ij}$$

Gear Ratio

$$\pm\frac{r_i}{r_j} = \frac{\omega_{1j} - \omega_{1k}}{\omega_{1i} - \omega_{1k}}$$

Gear Ratio is not equal to the speed ratio





$\omega_{arm}=\omega_{14}=40s^{-1}$ (CW)

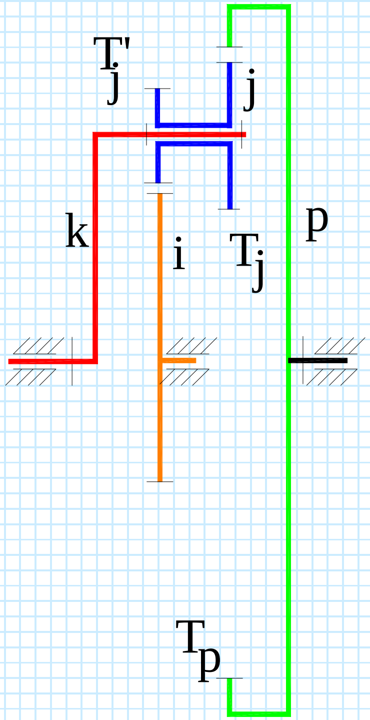
$\omega_{sun}=\omega_{12}=?$

$$\frac{\omega_{12}-\omega_{14}}{\omega_{11}-\omega_{14}}=-\frac{80}{20}\bullet\frac{20}{40}=-2$$

$\omega_{11}=0$

$$\omega_{12}=\omega_{14}-2(-\omega_{14})=3\omega_{14}$$

$\omega_{sun}=\omega_{12}=120\ s^{-1}$ (CW)



$$R_{ij} = \frac{\omega_{ki}}{\omega_{ki}} = \frac{\omega_{1j} - \omega_{1k}}{\omega_{1i} - \omega_{1k}}$$

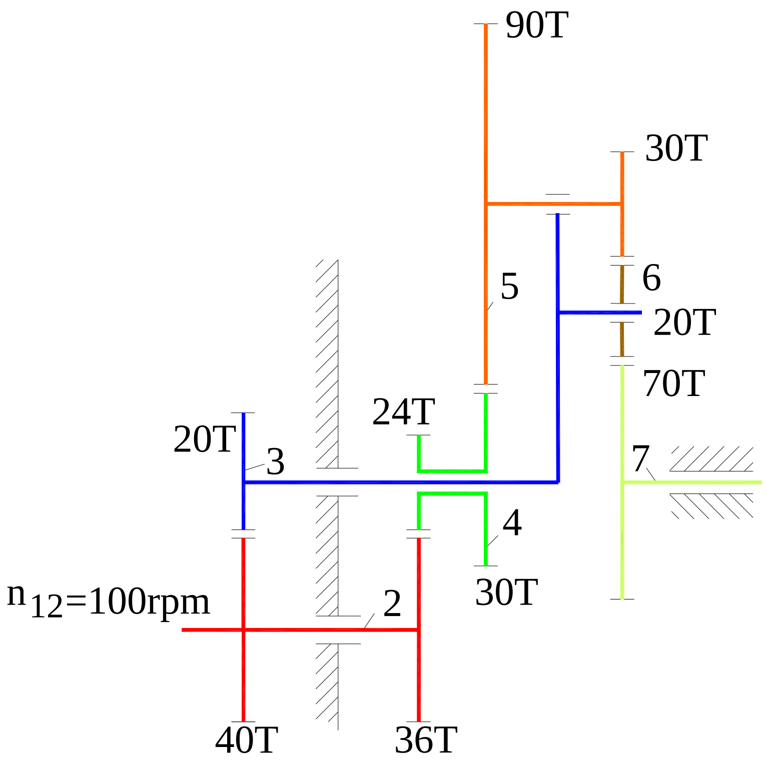
$$R_{pj} = \frac{\omega_{kj}}{\omega_{kp}} = \frac{\omega_{1j} - \omega_{1k}}{\omega_{1p} - \omega_{1k}} \qquad \frac{1}{R_{pj}} = R_{jp}$$

$$R_{ip} = \frac{R_{ij}}{R_{pj}} = \frac{\omega_{1p} - \omega_{1k}}{\omega_{1i} - \omega_{1k}}$$

$$R_{ip} = R_{ij} R_{jp} = (-1)^k \frac{T_j T_i}{T_j^/ T_p}$$

2-3 and 2-4 are simple gear trains

$$n_{14} = -\frac{36}{24} n_{12} \qquad n_{14} = -\frac{3}{2} n_{12}$$



$$n_{13} = -\frac{40}{20} n_{12} \qquad n_{13} = -2\, n_{12}$$

Now consider links 3,4,5,6 and 7.

Link 5 is the planet, link 3 is the arm

$$\frac{n_{17}-n_{13}}{n_{14}-n_{13}} = -\frac{30*30*20}{90*20*70} = -\frac{1}{7}$$

$$n_{17} = \frac{8}{7} n_{13} - \frac{1}{7} n_{14}$$

$$n_{17} = \frac{-8*2}{7} n_{12} - \frac{-3}{7*2} n_{12}$$

$$n_{17} = -\frac{29}{14} n_{12}$$

$$n_{17} = -\ 207.14 \text{ rpm}$$

n_{12}= 2000 rpm

n_{16}=?

First planet (arm red- link2)

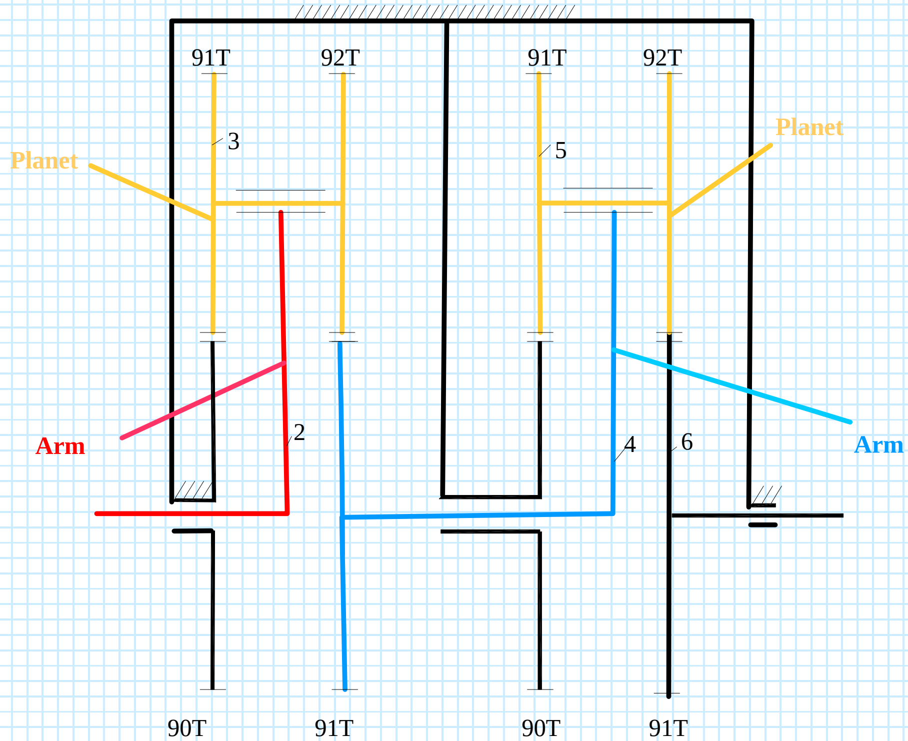
$$\frac{n_{14}-n_{12}}{n_{11}-n_{12}} = \frac{90*92}{91*91}$$

Since n_{11}=0 :

$$n_{14}= n_{12}-\frac{90*92}{91*91}n_{12}$$

$$n_{14}=\frac{8281-8280}{8281}n_{12}$$

$$n_{14}=\frac{1}{8281}n_{12}$$



Second planet (arm blue- link4)

$$\frac{n_{16}-n_{14}}{n_{11}-n_{14}} = \frac{90*92}{91*91}$$

$$n_{16}= n_{14}-\frac{90*92}{91*91}n_{14}$$

$$n_{16}=\frac{8281-8280}{8281}n_{14}$$

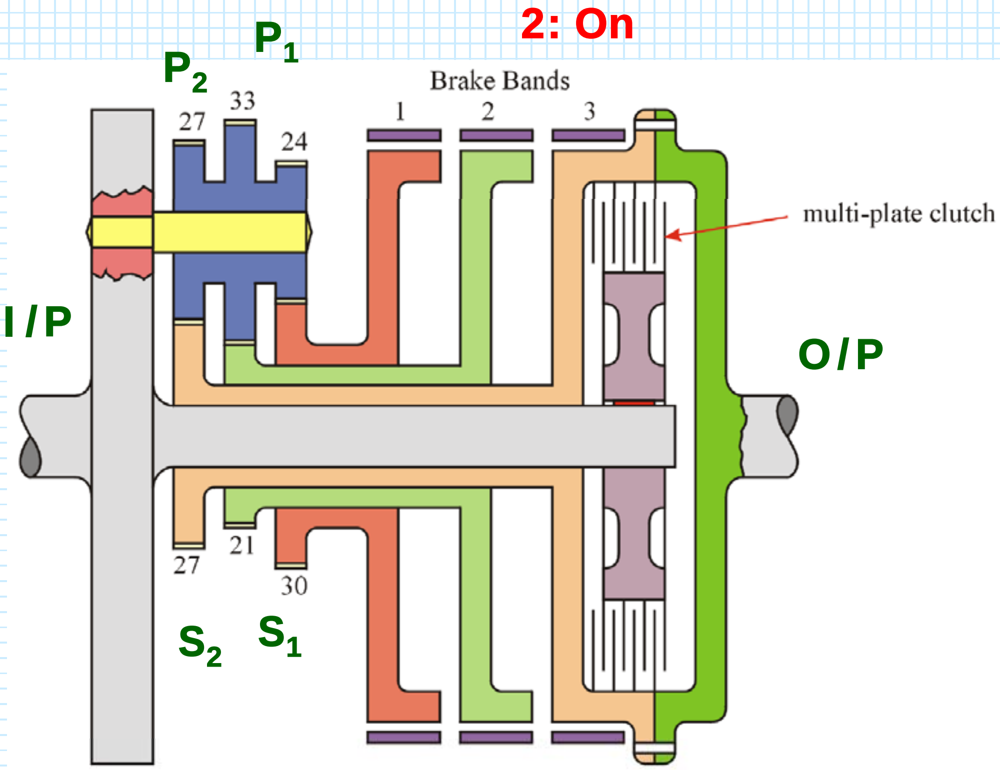
$$n_{16}=\frac{1}{8281}n_{14}$$

$$n_{16}=\frac{1}{8281^2}n_{12}$$

$$n_{16}=\frac{1}{68,574,961}n_{12}$$

n_{16}= 0.0000292 rpm

Example: Model T Ford gearbox



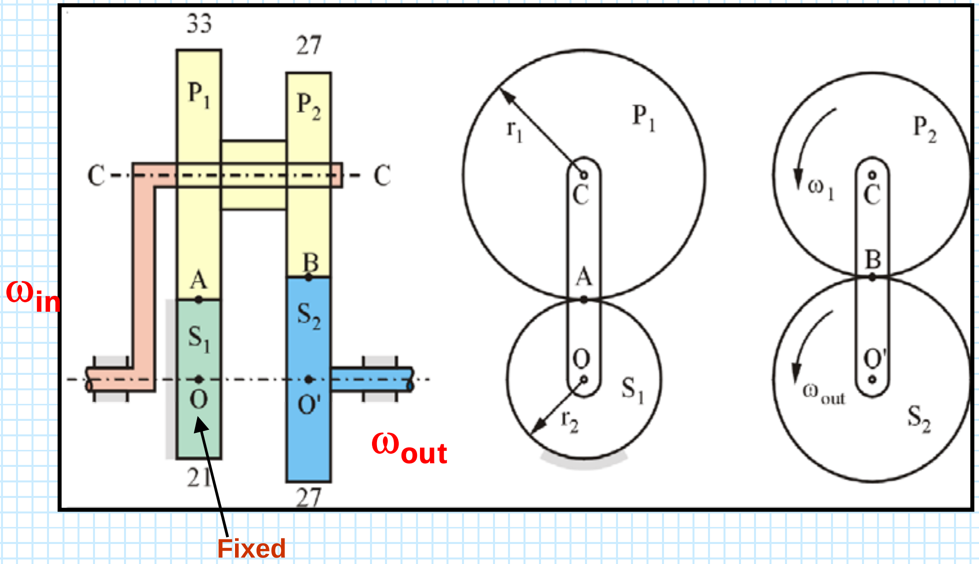
Gearbox : Integral with the engine. Foot operated 2 speed and reverse epicyclic transmission foot-brake, 1908 for 19 yrs

9 million were made!

http://www.t-ford.co.uk/car.htm

Gear	Clutch	Brake Bands 1	2	3	Gear Ratio
Idle	disengaged	off	off	on or off	—
Low	disengaged	off	on	off	?
High	engaged	off	off	off	1
Reverse	disengaged	on	off	off	?

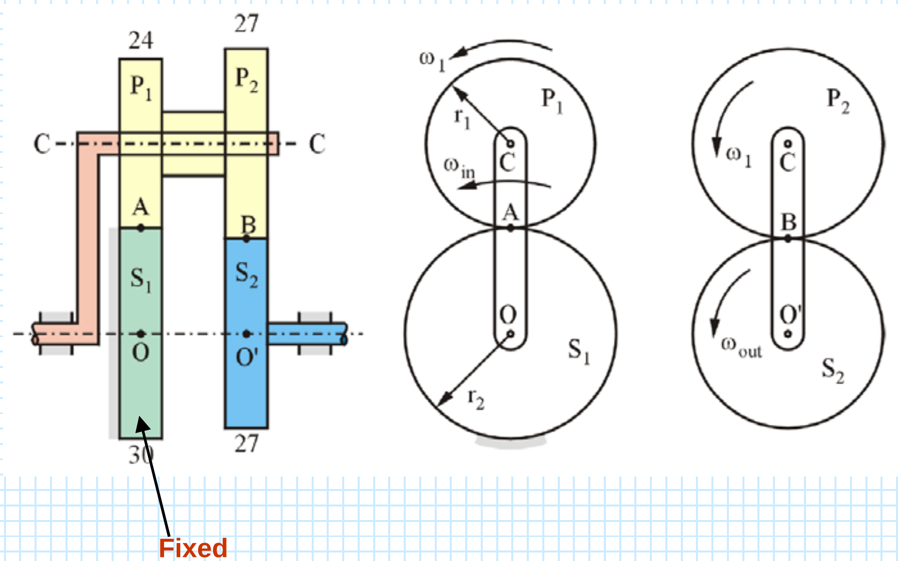
Low gear for the model T Ford



$$\frac{n_{out}-n_{in}}{n_{s1}-n_{in}} = \frac{21*27}{33*27} = 7/11$$

$$n_{out}=(1-7/11)\ n_{in} = 4/11 n_{in} = 0.3636\ n_{in}$$

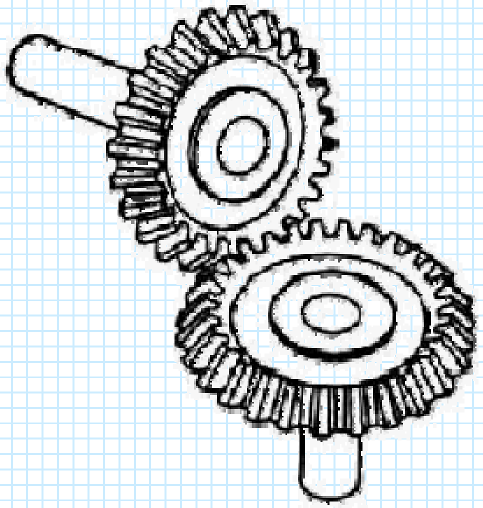
Reverse gear for the model T Ford



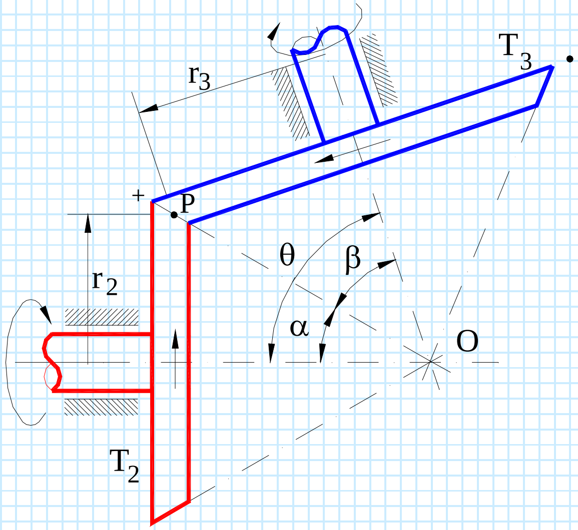
$$\frac{n_{out}-n_{in}}{n_{s1}-n_{in}} = \frac{30*27}{24*27} = 5/4$$

$$n_{out}=(1-5/4)\ n_{in} = -1/4 n_{in} = -\ 0.25\ n_{in}$$

GEAR TRAINS WITH BEVEL GEARS

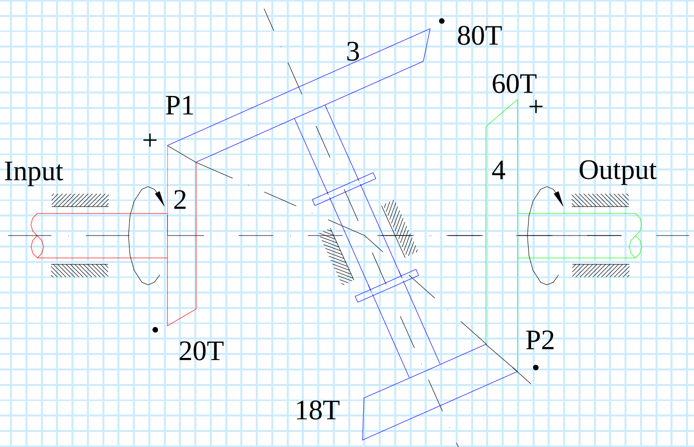


Bevel gears



$$\frac{\omega_{13}}{\omega_{12}} = \frac{r_2}{r_3} = \frac{T_2}{T_3} = \frac{(r_2 / OP)}{(r_3 / OP)}$$

$$= \frac{\sin\alpha}{\sin\beta} = R_{23}$$



Simple compound gear train
(axes of all gears are fixed axes)

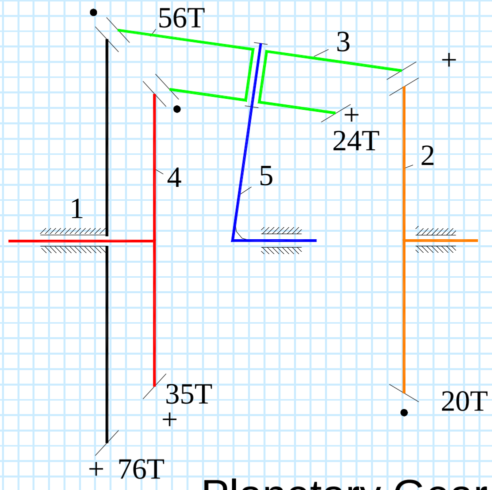
$$R_{23} = \frac{20}{80} = \frac{\omega_{13}}{\omega_{12}}$$

and

or $$R_{34} = \frac{18}{60} = \frac{\omega_{14}}{\omega_{13}}$$

$$R_{24} = \frac{\omega_{14}}{\omega_{12}} = \frac{20*18}{80*60} = \frac{3}{40} = \frac{\text{Product of driving gear tooth number}}{\text{Product of driven gear tooth number}}$$

$$R_{24} = \frac{\omega_{14}}{\omega_{12}} = +\frac{3}{40} = +0.075$$



Planetary Gear Train

$$R_{12} = \frac{n_{12} - n_{15}}{n_{11} - n_{15}} = -\frac{76*56}{56*20} = -\frac{19}{5}$$

Since n_{11}=0: $$n_{15} = \frac{5}{24} n_{12}$$

$$R_{14} = \frac{n_{14} - n_{15}}{n_{11} - n_{15}} = +\frac{76*24}{56*35} = +\frac{228}{245}$$

Since n_{11}=0:

$$n_{14} = \left(1 - \frac{228}{245}\right) n_{15} = \frac{17}{245} n_{15} = \frac{17}{245} * \frac{5}{24} n_{12}$$

$$N_{24} = \frac{n_{14}}{n_{12}} = 0.0145$$

Motion from 2 to 3 and 2 to 4 are simple gear trains (axes fixed):

$$n_{14} = \frac{50}{20} n_{12}$$

$$n_{13} = \frac{50}{20} n_{12}$$

Links 3 and 4 rotate in different directions

$$n_{13} = -n_{14} = 2.5\, n_{12}$$

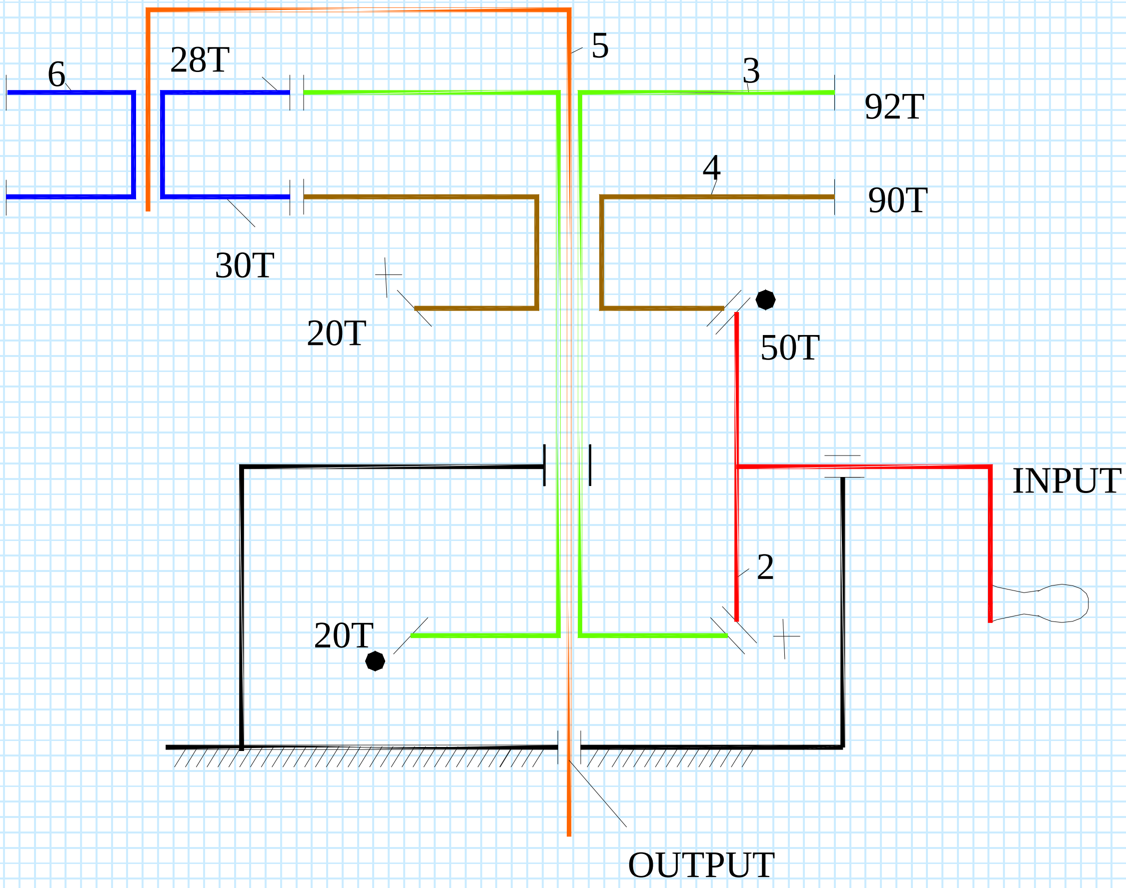
Considering links 3,4,5 and 6; link 6 is the planet and link 5 is the arm (output)

$$\frac{n_{13}-n_{15}}{n_{14}-n_{15}} = +\frac{90*28}{30*92}$$

$$n_{13}-n_{15} = \frac{21}{23}(n_{14}-n_{15})$$

$$\frac{2}{23} n_{15} = n_{13} - \frac{21}{23} n_{14}$$

$$\frac{2}{23} n_{15} = \frac{5}{2}\,\frac{44}{23}\, n_{12}$$



$$n_{15} = 55\, n_{12}$$