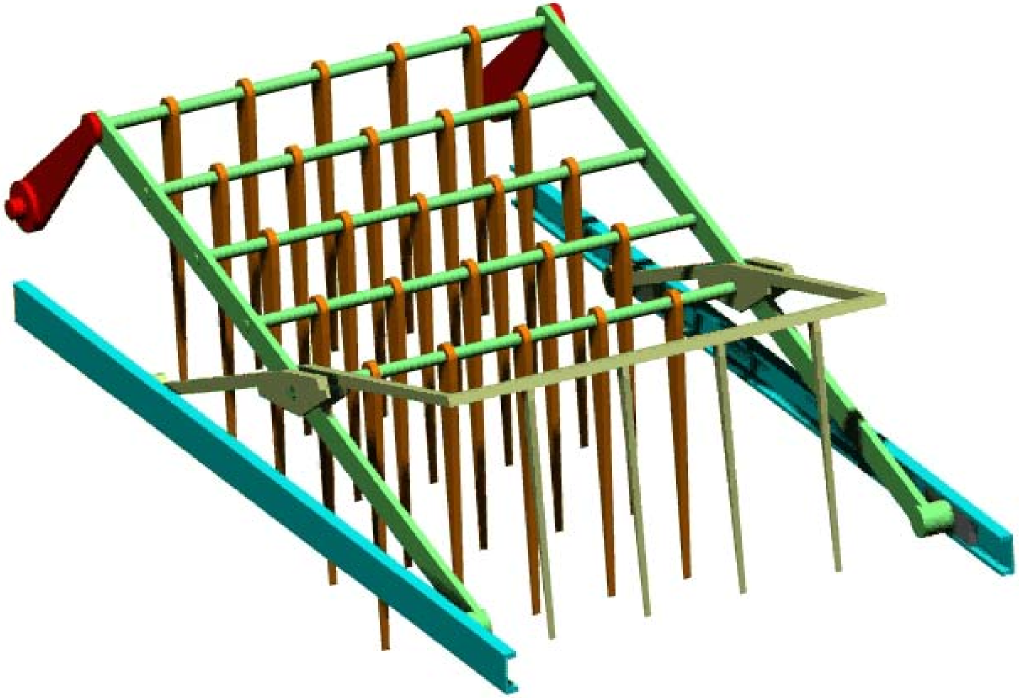
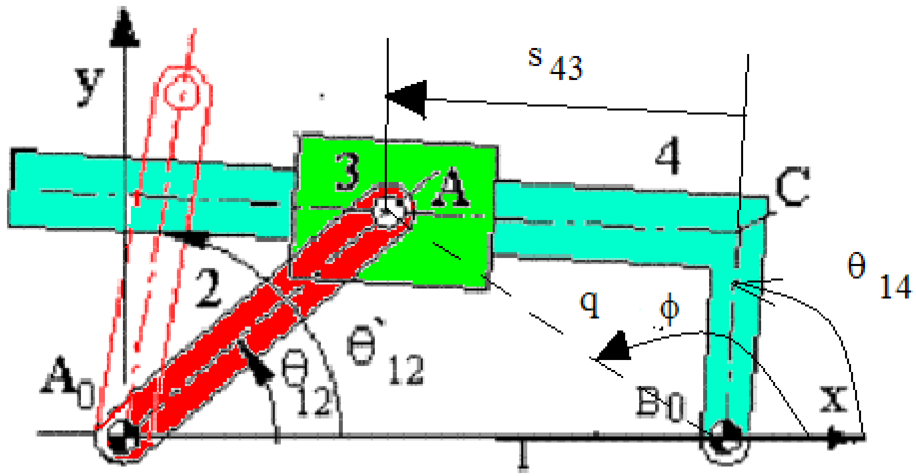


ME301 THEORY OF MACHINES 1- MECHANISMS



B_0C is perpendicular to the slider axis

Inverted Slider Crank

$A_0A=a_2$, $A_0B_0=a_1$, $B_0C=a_4$

$s_x=-a_1+a_2\cos(\theta_{12})$

$s_y= a_2\sin(\theta_{12})$

$q= \mathrm{mag}(s_x,s_y)$

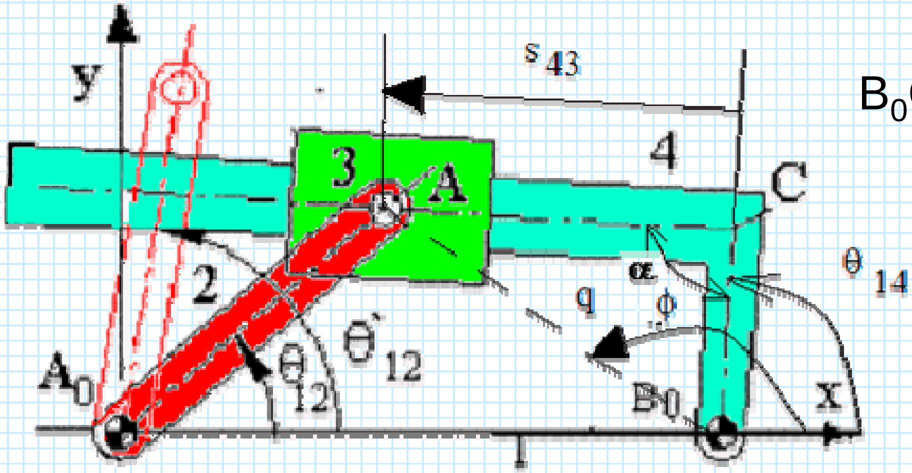
$\phi = \tan^{-1}(s_x,s_y)$

$$s_{43}=\sqrt{q^2-a_4^2}$$

$$\eta=\sin^{-1}\left(\frac{s_{43}}{q}\right)=\cos^{-1}\left(\frac{c}{q}\right)=\tan^{-1}\left(\frac{s_{43}}{c}\right)$$

$$\theta_{14}=\phi\pm\eta$$

```
Function InvSliderCrank2(Crank, Fixed, Eccentricity,
Config, Theta)
Dim A(2)
sX = Crank * Cos(Theta) - Fixed
sY = Crank * Sin(Theta)
q = Mag(sX, sY)
Fi = Ang(sX, sY)
S43 = Sqr(q ^ 2 - Eccentricity ^ 2)
Eta = Acos(Eccentricity / q)
A(0) = S43
A(1) = Fi - Config * Eta
InvSlider2 = A
End Function
```



B_0C is not perpendicular to the slider axis

$A_0A=a_2$, $A_0B_0=a_1$, $B_0C=a_4$

$s_x=-a_1+a_2\cos(\theta_{12})$

$s_y= a_2\sin(\theta_{12})$

$q= \text{mag}(s_x,s_y)$

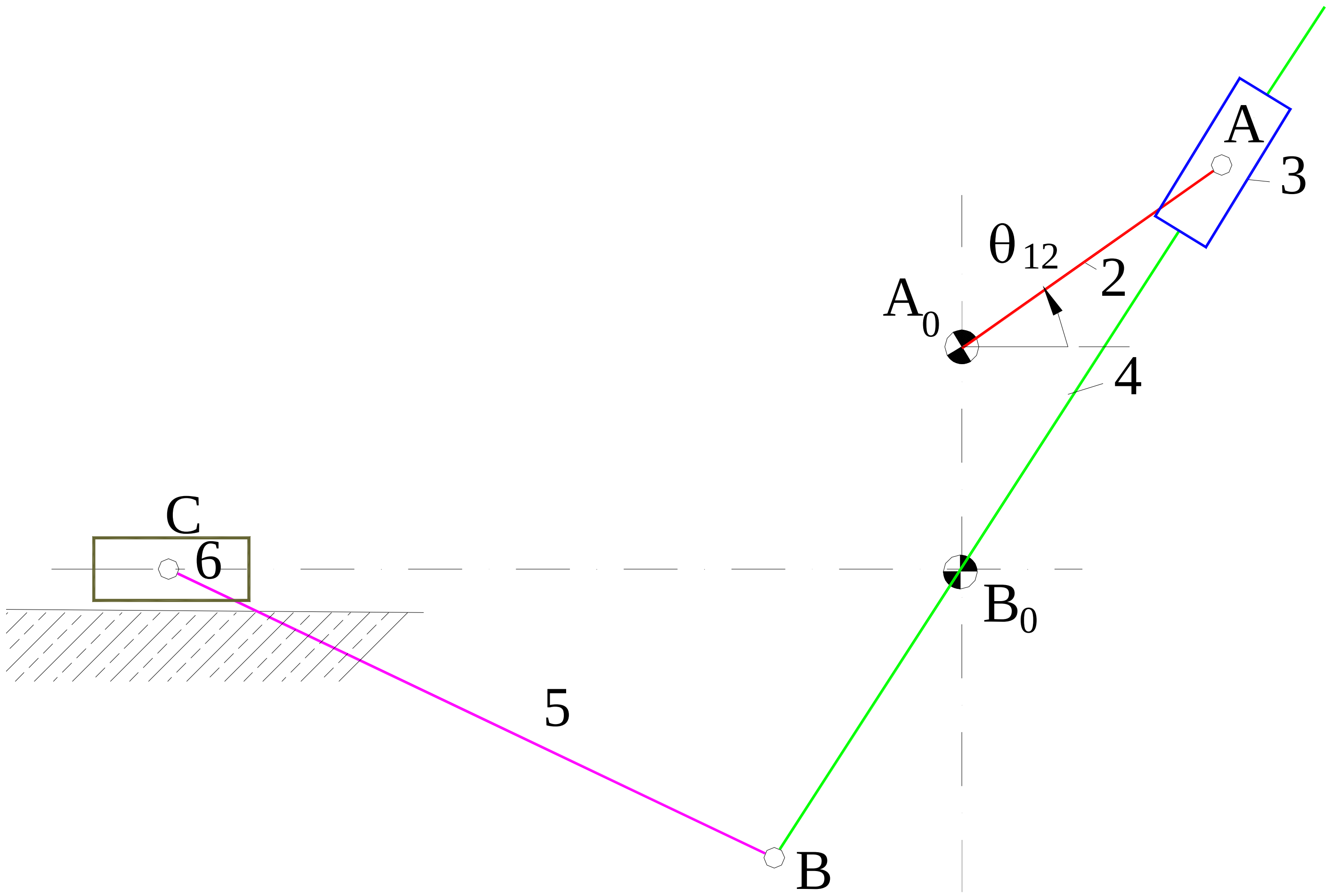
$\phi = \tan^{-1}(s_x,s_y)$

$$s_{43} = -a_4\cos\alpha + \sqrt{q^2 - a_4^2\sin^2\alpha}$$

$$\eta = \tan^{-1}(a_4 + s_{43}\cos\alpha, s_{43}\sin\alpha)$$

$$\theta_{14} = \phi \pm \eta$$

```
Function InvSlider2(Crank, Fixed, Eccentricity, Alfa,
Config, Theta)
'Determines the slider displacement and the output
link angle for an inverted slider-crank mechanism
Dim A(2) As Double
Dim S, Fi, Si, Q, Delta As Double
Dim sx, sy As Double
sx = -Fixed + Crank * Cos(Theta)
sy = Crank * Sin(Theta)
S = Mag(sx, sy)
Fi = Ang(sx, sy)
Delta = S ^ 2 - Eccentricity ^ 2 * Sin(Alfa) ^ 2
Q = -Eccentricity * Cos(Alfa) + Sqr(Delta)
Si = Ang((Eccentricity + Q * Cos(Alfa)), Q * Sin(Alfa))
A(0) = Q
A(1) = Fi - Config * Si
InvSlider2 = A
End Function
```



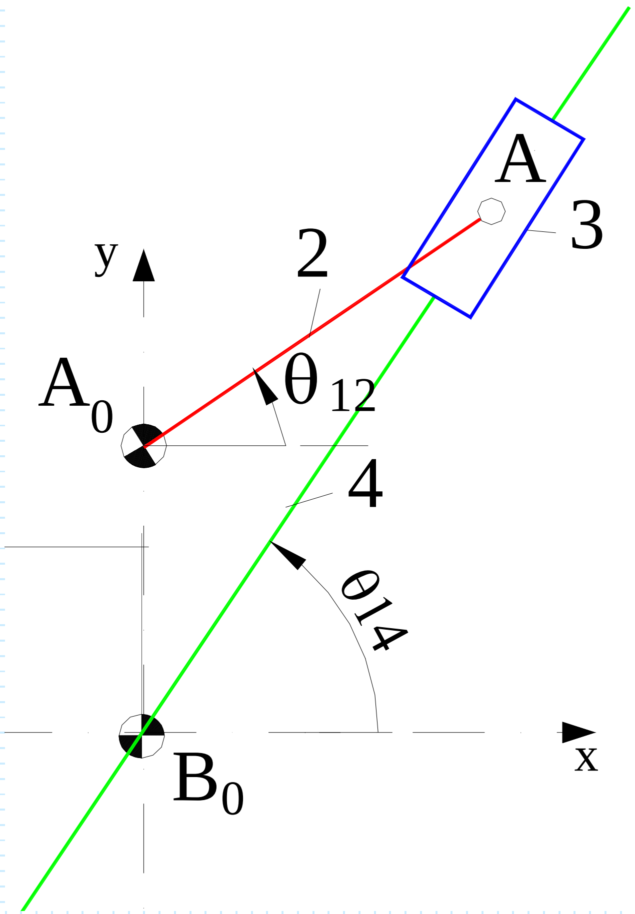
Example

Given:

All the necessary link dimensions and the input crank angle θ_{12}

Determine:

All the position variables and the displacement of the slider 6 as a function of input crank angle



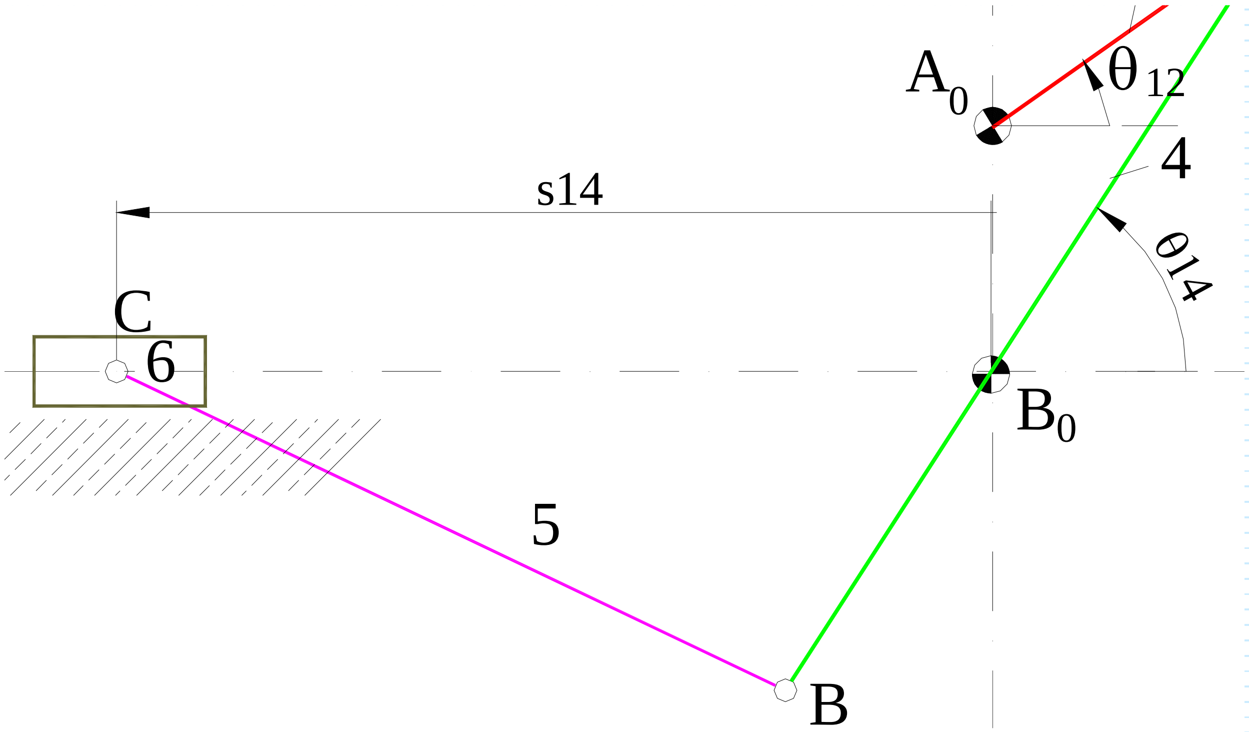
1. Consider the loop formed by links 1,2,3 and 4. An inverted slider-crank mechanism.

θ_{14}= {InvSlider(a_2,a_1,0,1, $\theta_{12}+\pi/2$)-$\pi/2$}

$x_a=a_2\cos\theta_{12}$, $y_a=a_1+a_2\sin\theta_{12}$

$$s_{43} = mag(x_a, y_a)$$

$$\theta_{14} = \tan^{-1}(x_a, y_a)$$



2. Consider the loop 1,4,5,6. A slider Crank Mechanism

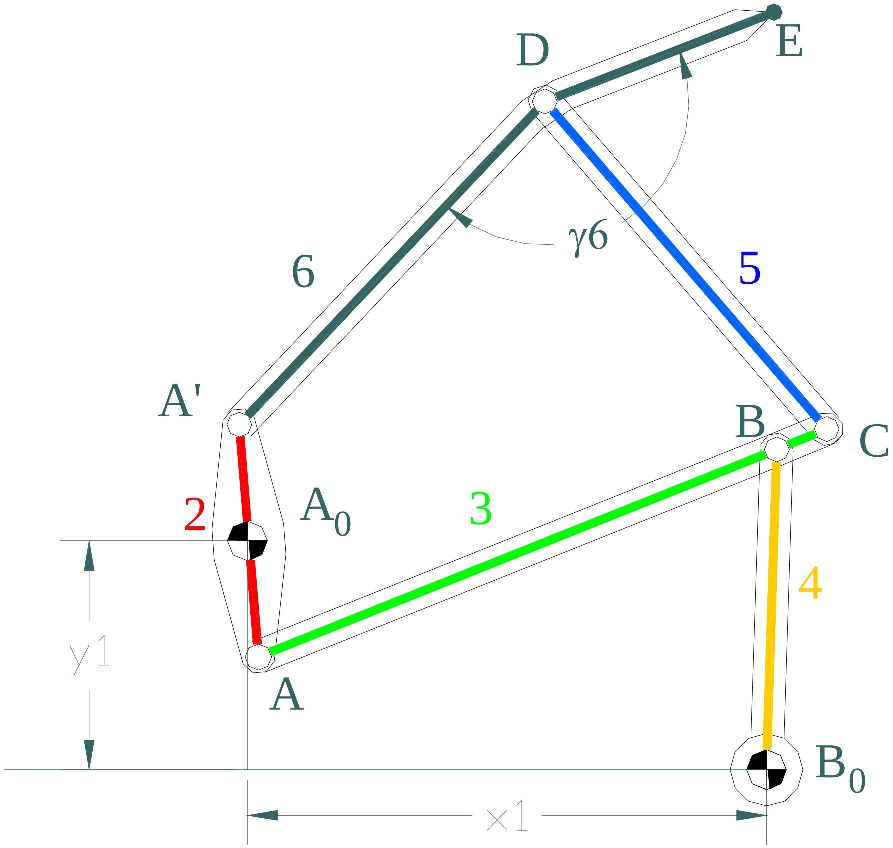
s_{14}=-{SliderCrank(a_4,a_5,0,1,θ_{14})}

$x_b=-a_4\cos(\theta_{14})$, $y_b=-a_4\sin(\theta_{14})$

$x_c=s_{14}$, $y_c=0$

Example:

Kinematic Analysis of an Oat Baler Mechanism



Given: All the necessary link length dimensions are known.
Let:
$a_2 = A_0A = A_0A'$
$a_3 = AB$, $b_3 = BC$
$a_4 = B_0B$
$a_5 = CD$
$a_6 = A'D$
$b_6 = DE$

Also, as shown on the figure, the fixed dimensions x_1, y_1 and the fixed angle γ_6 are known.

Determine:

A complete position analysis of the mechanism

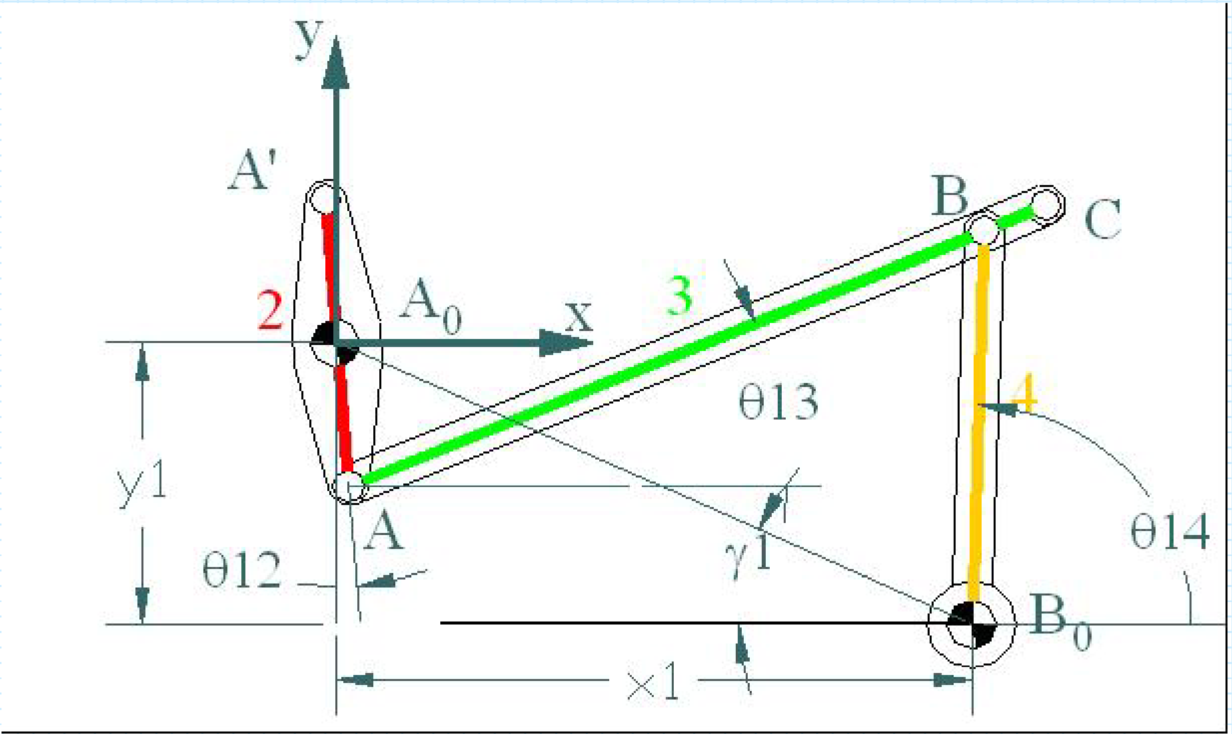
1. Solve the four-bar Loop (links 1,2,3,4)

$$a_1 = \sqrt{x_1^{\;2} + y_1^{\;2}}$$

and

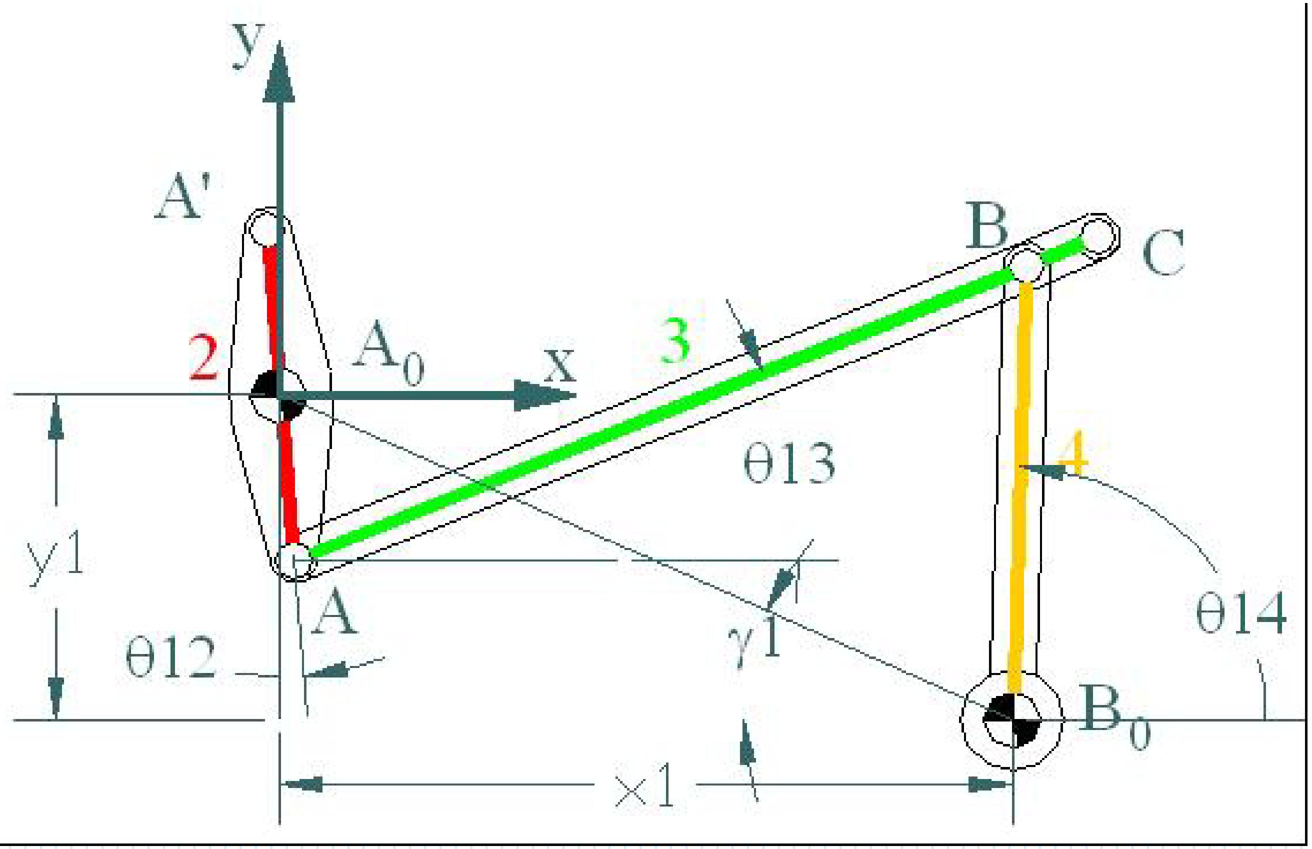
$$\gamma_1 = \tan^{-1}\left(y_1 / x_1\right)$$

$\theta_{12}' = \theta_{12} + \gamma_1 - \pi/2.$



θ_{13} and θ_{14} will be determined using the function fourBar2() in the form:

$\{FourBar2(a_2,a_3,a_4,a_1,1,\theta_{12}')- \gamma_1\}$



$$x_a = a_2 \sin(\theta_{12})$$

$$y_a = -a_2 \cos(\theta_{12})$$

$$x_{a'} = -a_2 \sin(\theta_{12}) = -x_a$$

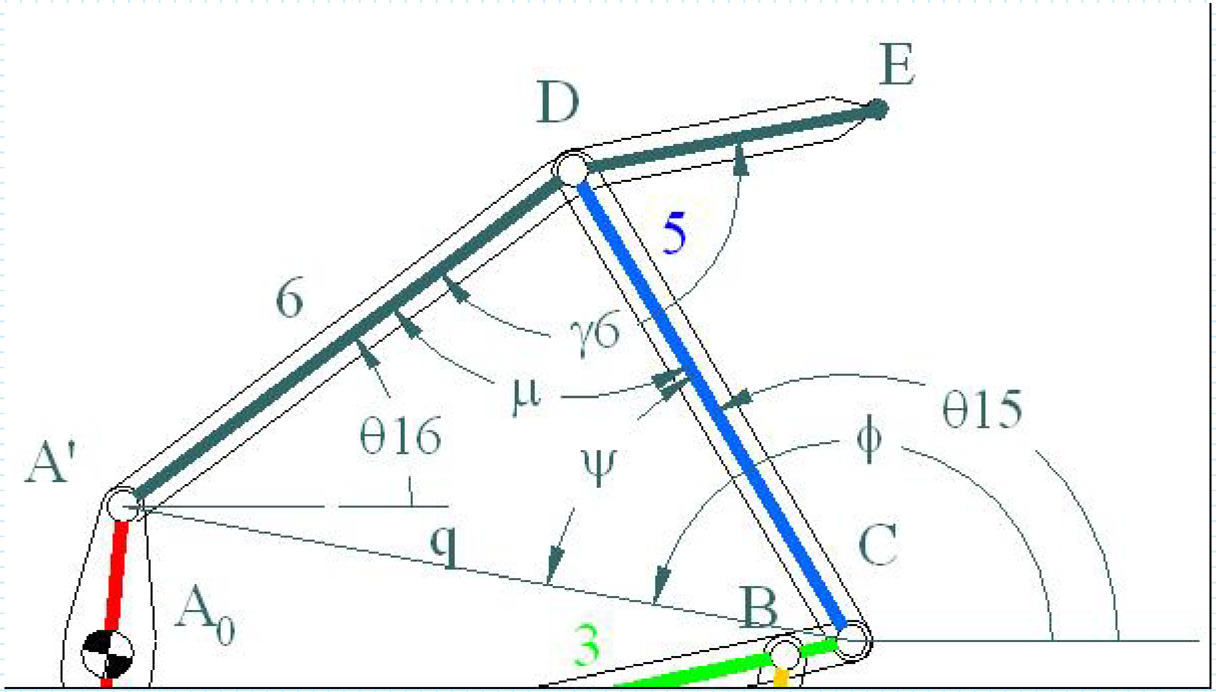
$$y_{a'} = a_2 \cos(\theta_{12}) = -y_a$$

$$x_b = x_1 + a_4 \cos(\theta_{14})$$

$$y_b = -y_1 + a_4 \sin(\theta_{14})$$

$$x_c = a_2 \sin(\theta_{12}) + (a_3 + b_3)\cos(\theta_{13}) = x_a + (a_3 + b_3)\cos(\theta_{13})$$

$$y_c = -a_2 \cos(\theta_{12}) + (a_3 + b_3)\sin(\theta_{13}) = y_a + (a_3 + b_3)\sin(\theta_{13})$$



$$q = \sqrt{(x_{a'} - x_c)^2 + (y_{a'} - y_c)^2}$$

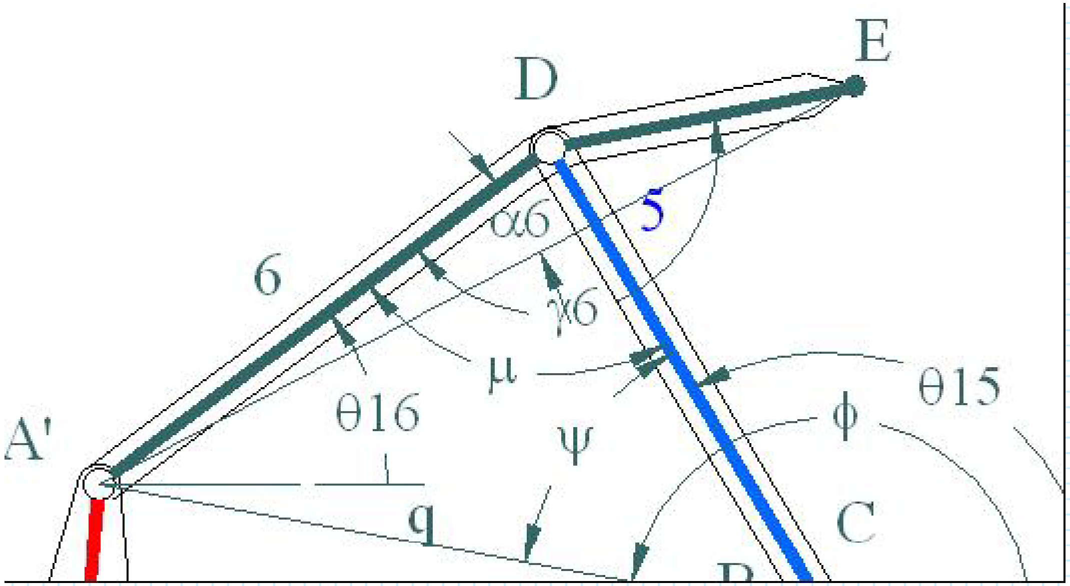
$$\phi = \tan^{-1}((x_{a'} - x_c), (y_{a'} - y_c))$$

$$\psi = \cos^{-1}\left[\frac{q^2 + a_5^2 - a_6^2}{2qa_5}\right]$$

$$\mu = \cos^{-1}\left[\frac{a_6^2 + a_5^2 - q^2}{2a_6a_5}\right]$$

$\theta_{15} = \phi - \psi$

$\theta_{16} = \theta_{15} - \mu$



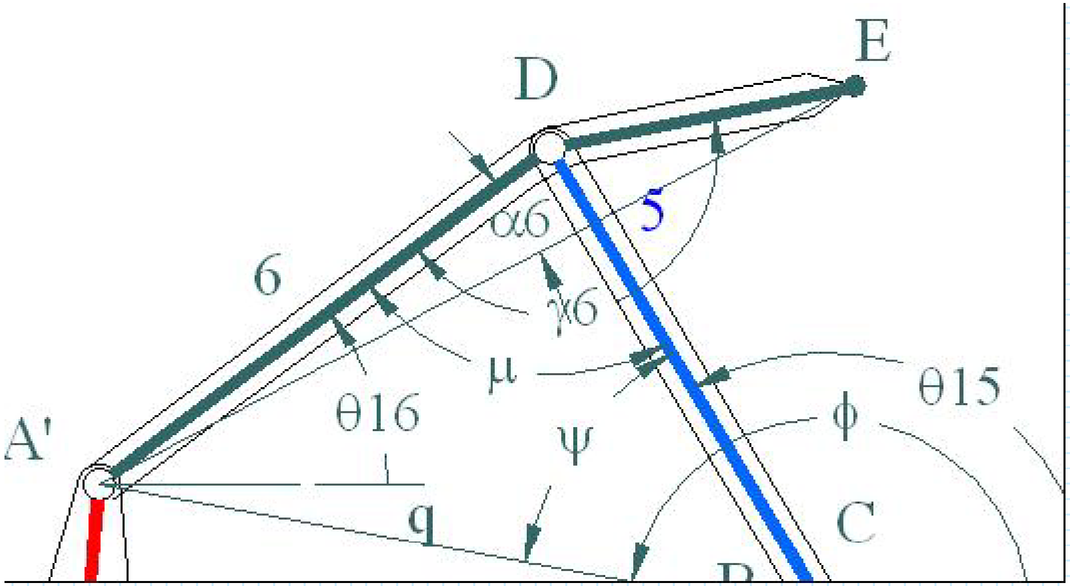
$$x_d = x_{a'} + a_6 \cos(\theta_{16})$$
$$y_d = y_{a'} + a_6 \sin(\theta_{16})$$

or

$$x_d = x_c + a_5 \cos(\theta_{15})$$
$$y_d = y_c + a_5 \sin(\theta_{15})$$

and

$$x_e = x_d + b_6 \cos(\theta_{16} - \pi + \gamma_6) = x_d - b_6 \cos(\theta_{16} + \gamma_6)$$
$$y_e = x_d + b_6 \sin(\theta_{16} - \pi + \gamma_6) = x_d - b_6 \sin(\theta_{16} + \gamma_6)$$



Or Consider Triangle A'DE on link 6

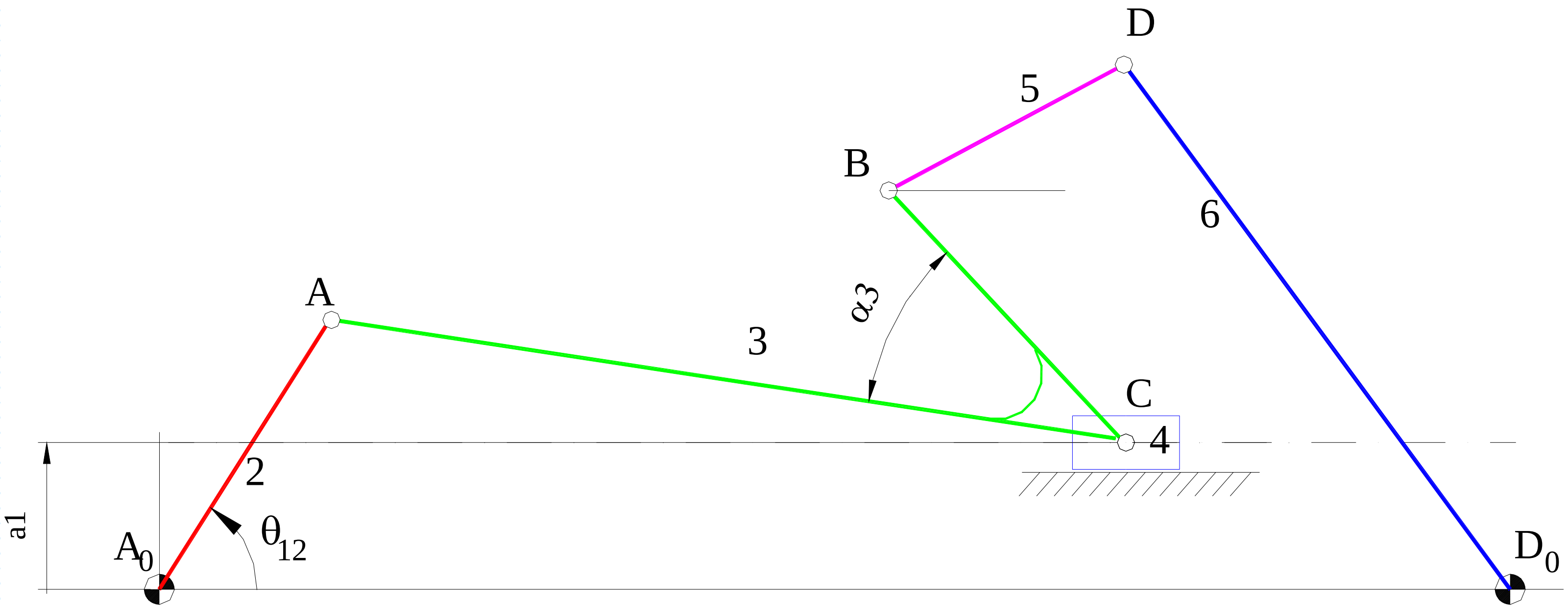
$$c_6 = \sqrt{(a_6{}^2 + b_6{}^2 - 2a_6 b_6 \cos(\gamma_6))}$$

$$\alpha_6 = \cos^{-1}\left[\frac{a_6^2 + c_6^2 - b_6^2}{2a_6 c_6}\right]$$

$$x_e = x_{a'} + c_6 \cos(\theta_{16} - \alpha_6)$$

$$y_e = y_{a'} + c_6 \sin(\theta_{16} - \alpha_6)$$

Example:



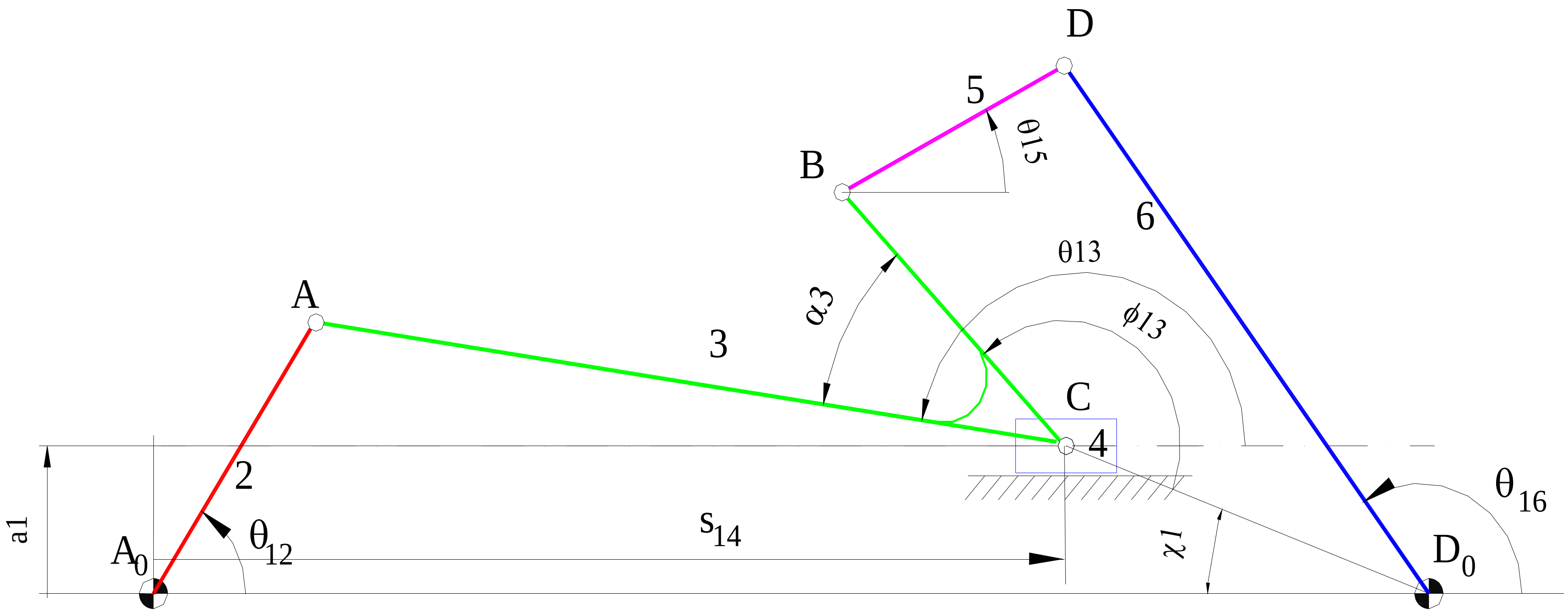
Given:

All the necessary link length dimensions.

$A_0A=a_2$, $AC=a_3$, $BC=b_3$, $BD=a_5$, $D0D=a_6$, $A_0D_0=b_1$, $<ACD=\alpha_3$

Determine:

For any input crank angle θ_{12}, all the position parameters



1. Obtain θ_{13}, s_{14} from the SliderCrank2() function

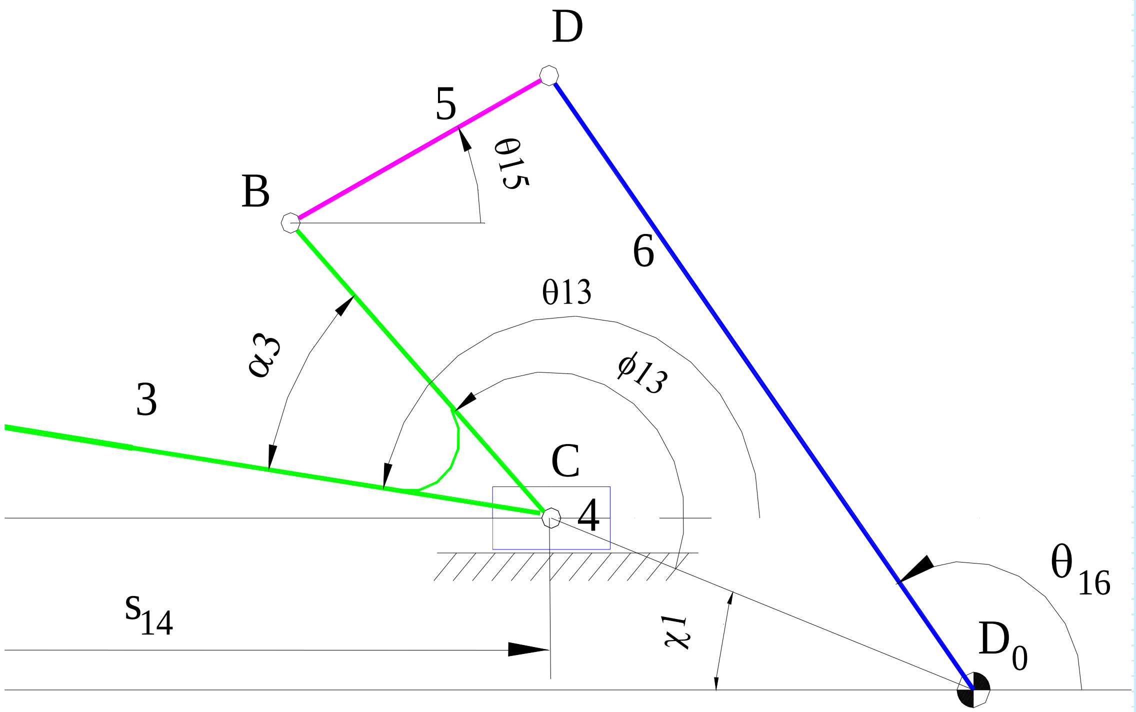
   $[\theta_{13}, s_{14}]$= SliderCrank2(a_2,a_3,a_1,1,θ_{12})

$$x_a = a_2 \cos\theta_{12}$$

$$y_a = a_2 \sin\theta_{12}$$

$$x_b = s_{14} + b_3 \cos(\theta_{13} - \alpha_3)$$

$$y_b = b_3 \sin(\theta_{13} - \alpha_3) + a_1$$



$$D_0C = S = \sqrt{(b_1 - s_{14})^2 + a_1^{\,2}}$$

$$\chi_1 = \tan^{-1}\left((b_1 - s_{14}), a_1\right)$$

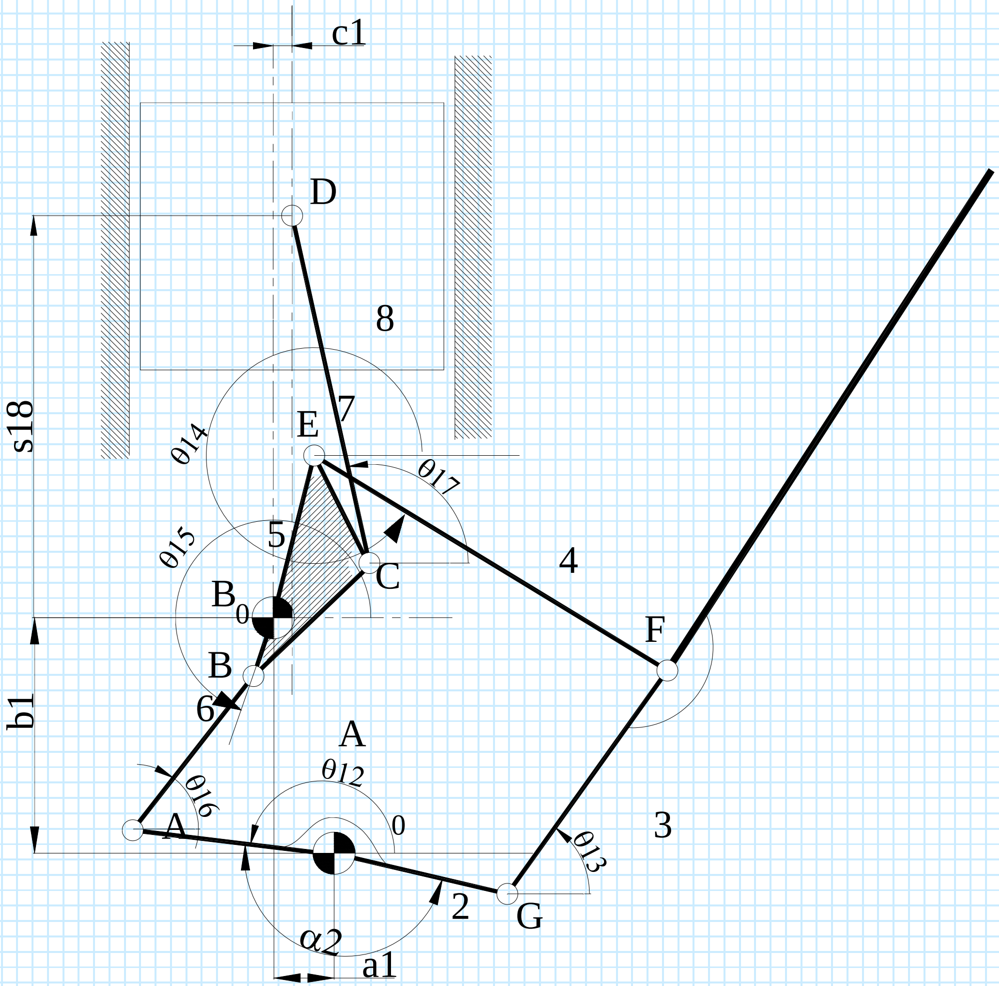
$$\phi_{13} = \theta_{13} + \chi_1 - \alpha_3$$

2. Obtain θ_{15}, θ_{16} using FourBar2() function

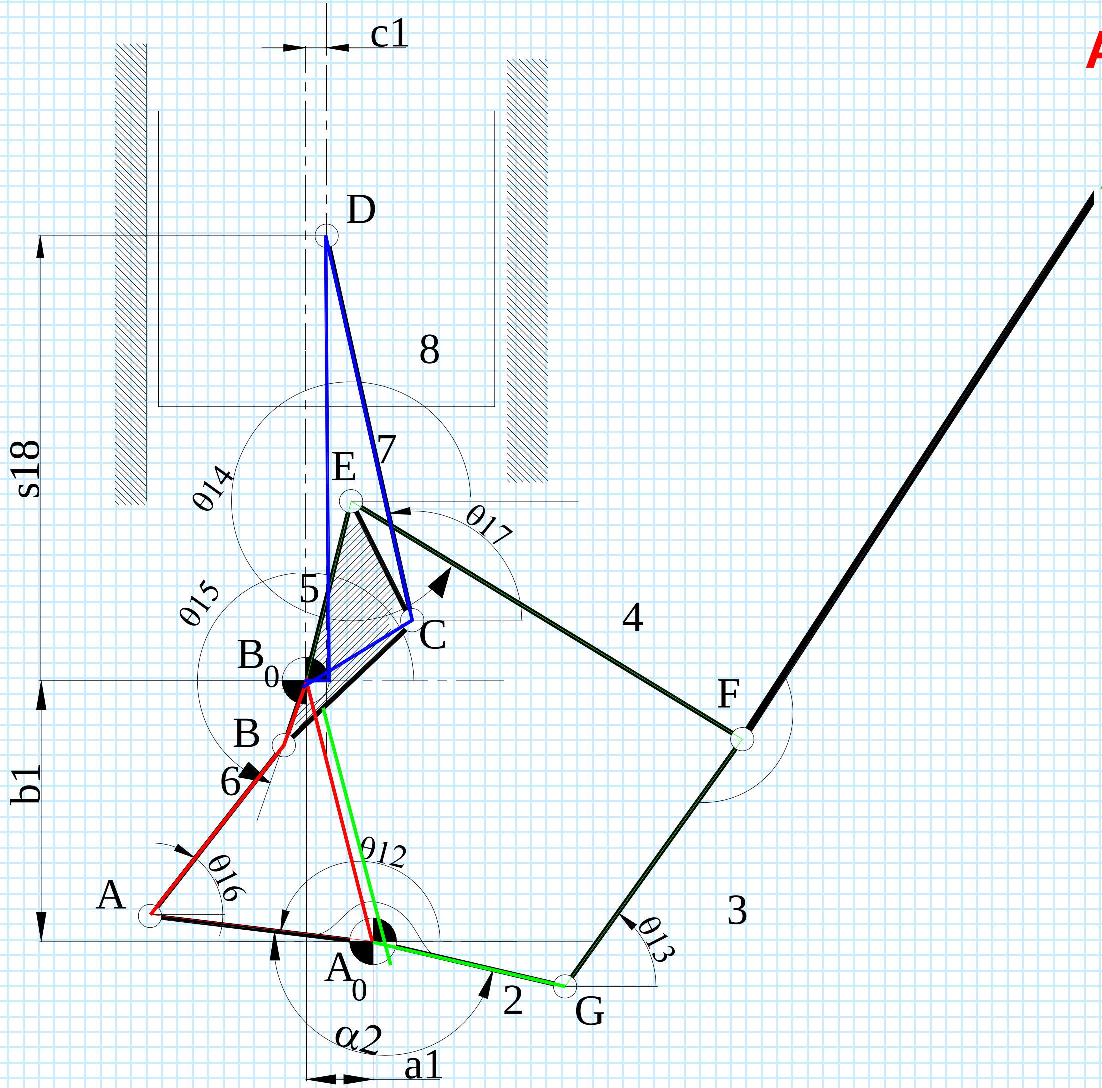
$[\theta_{15}, \theta_{16}]$= {FourBar2($b_3$,$a_5$,$a_6$,S, 1,$\phi_{13}$)-$\chi_1$}

$$x_d = b_1 + a_6\cos(\theta_{16})$$

$$y_d = a_6\sin(\theta_{16})$$



$A_0A=a_{2,}$; $A_0G=b_2$; $GF=a_3$; $BA=a_6$;
$B_0B=a_5$ $B_0C=b_5$; $B_0E=c_5$; $EF=a_5$;
$CD=a_5$; $<BB_0C=\alpha_5$; $<BB_0E=\beta_5$



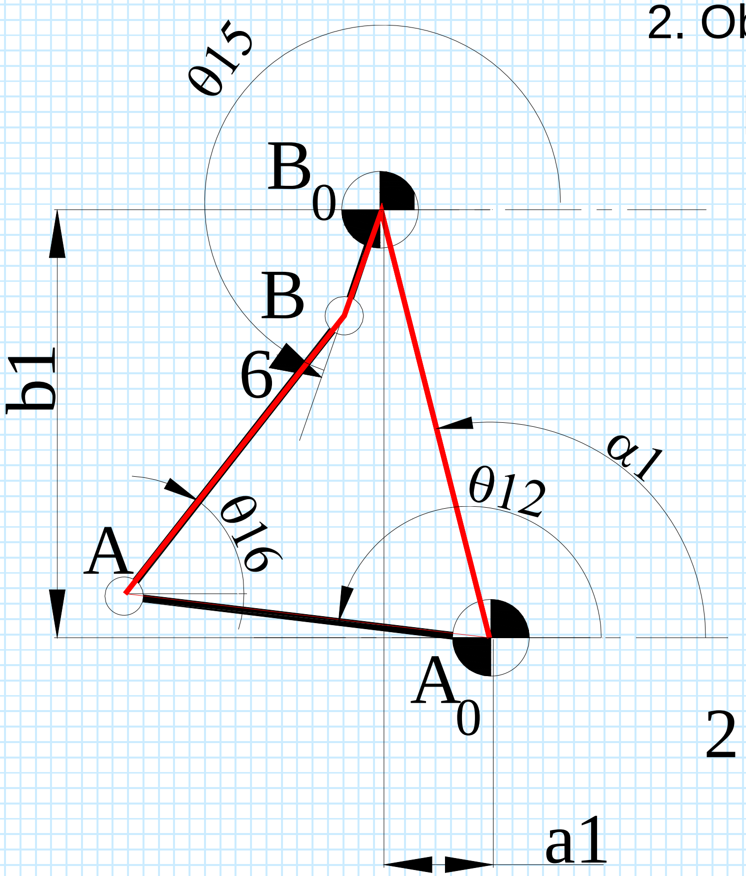
$A_0ABB_0A_0$ loop: A four-bar

$A_0GFEB_0A_0$ loop

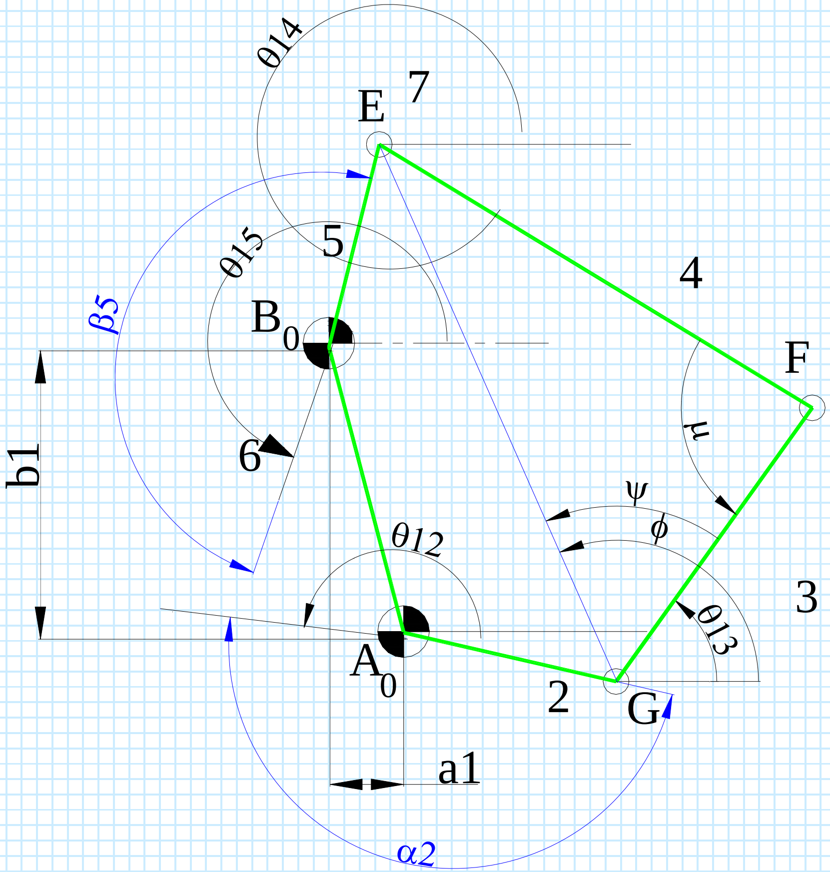
B_0CDB_0 loop: SliderCrank

2. Obtain θ_{15}, θ_{16} using FourBar2() function

$[\theta_{16}, \theta_{15}]= \{FourBar2(a_2,a_6,a_6,d_1, 1,(\theta_{12}-\alpha_1))+\alpha_1\}$



$d_1 = \sqrt{a_1^2 + b_1^2}$; $\alpha_1 = \tan^{-1}\left[-a_1; b_1\right]$ (note the quadrant!!)



$$x_G = b_2 \cos(\theta_{12} + \alpha_2)$$

$$y_G = b_2 \sin(\theta_{12} + \alpha_2)$$

$$x_E = -a_1 + c_5 \cos(\theta_{15} - \beta_5)$$

$$y_E = b_1 + c_5 \sin(\theta_{15} - \beta_5)$$

$$|GE| = s = \sqrt{\left[(x_E - x_G)^2 + (y_E - y_G)^2\right]}$$

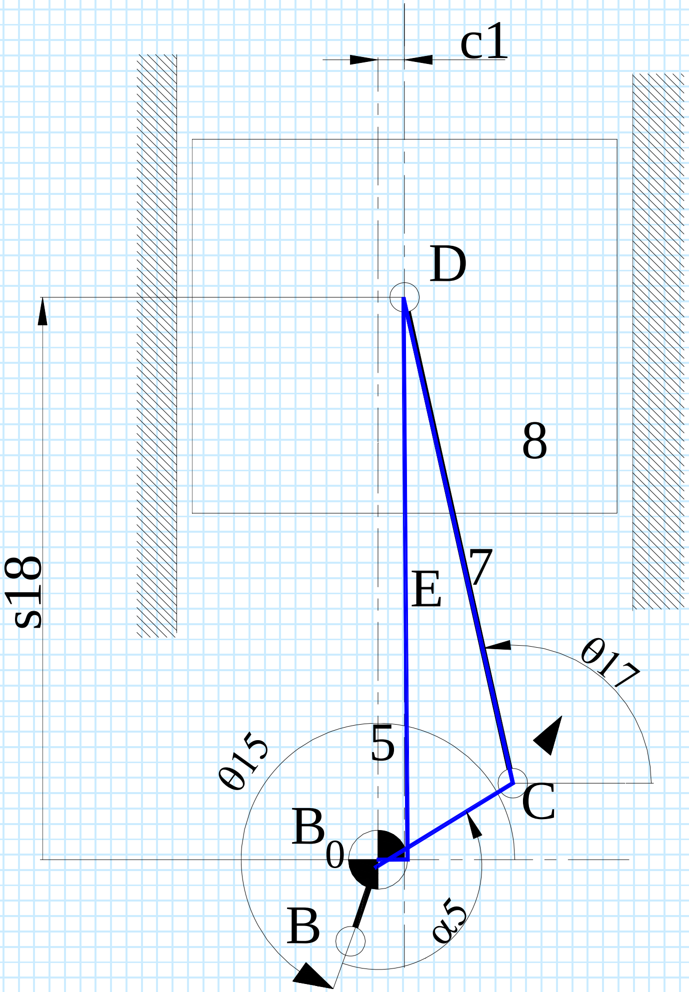
$$\phi = \tan^{-1}\left[(x_E - x_G);(y_E - y_G)\right]$$

$$\psi = ang\cos(s; a_3; a_4)$$

$$\mu = ang\cos(a_4; a_3; s)$$

$$\theta_{13} = \phi - \psi$$

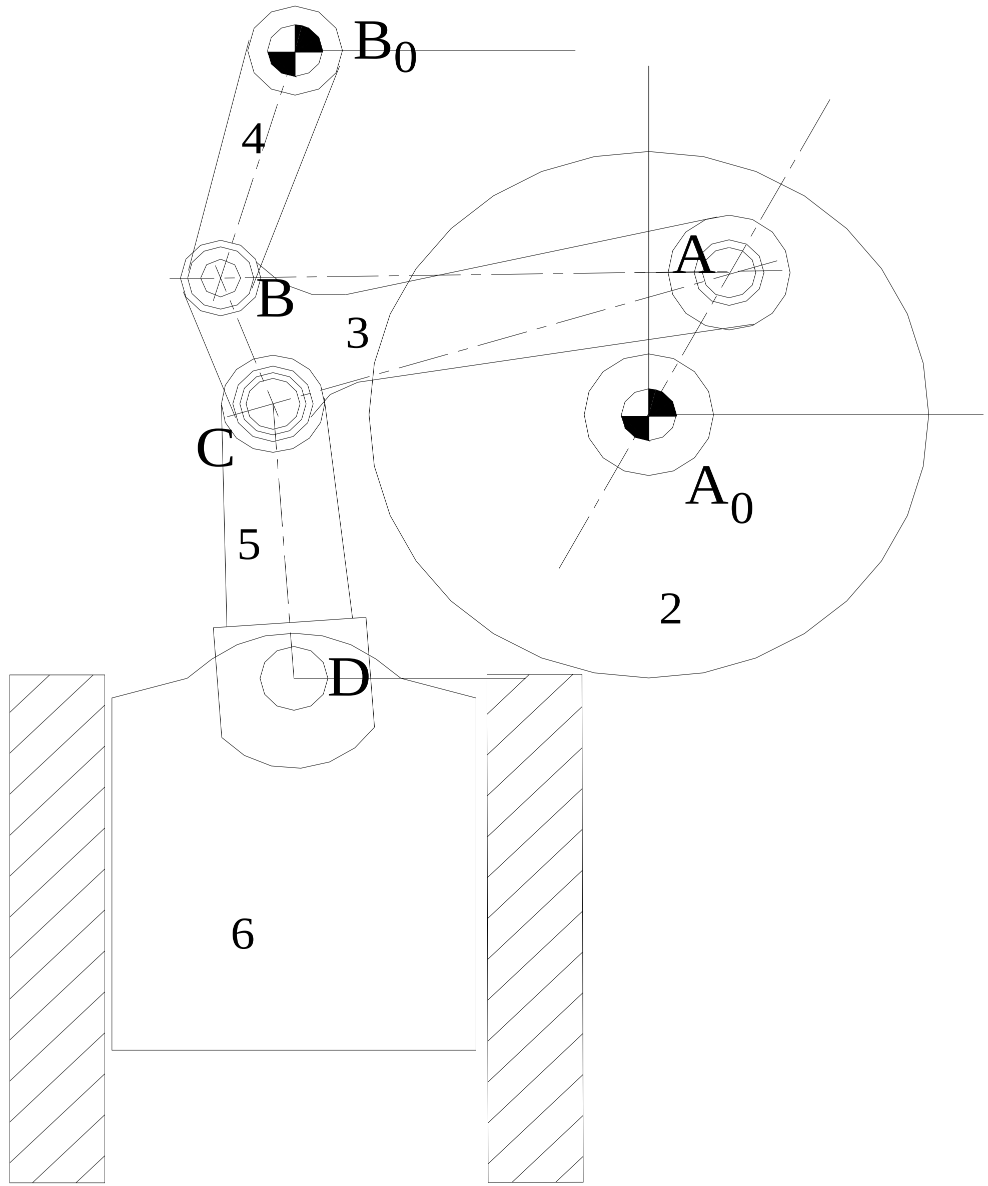
$$\theta_{14} = \theta_{13} - \mu$$

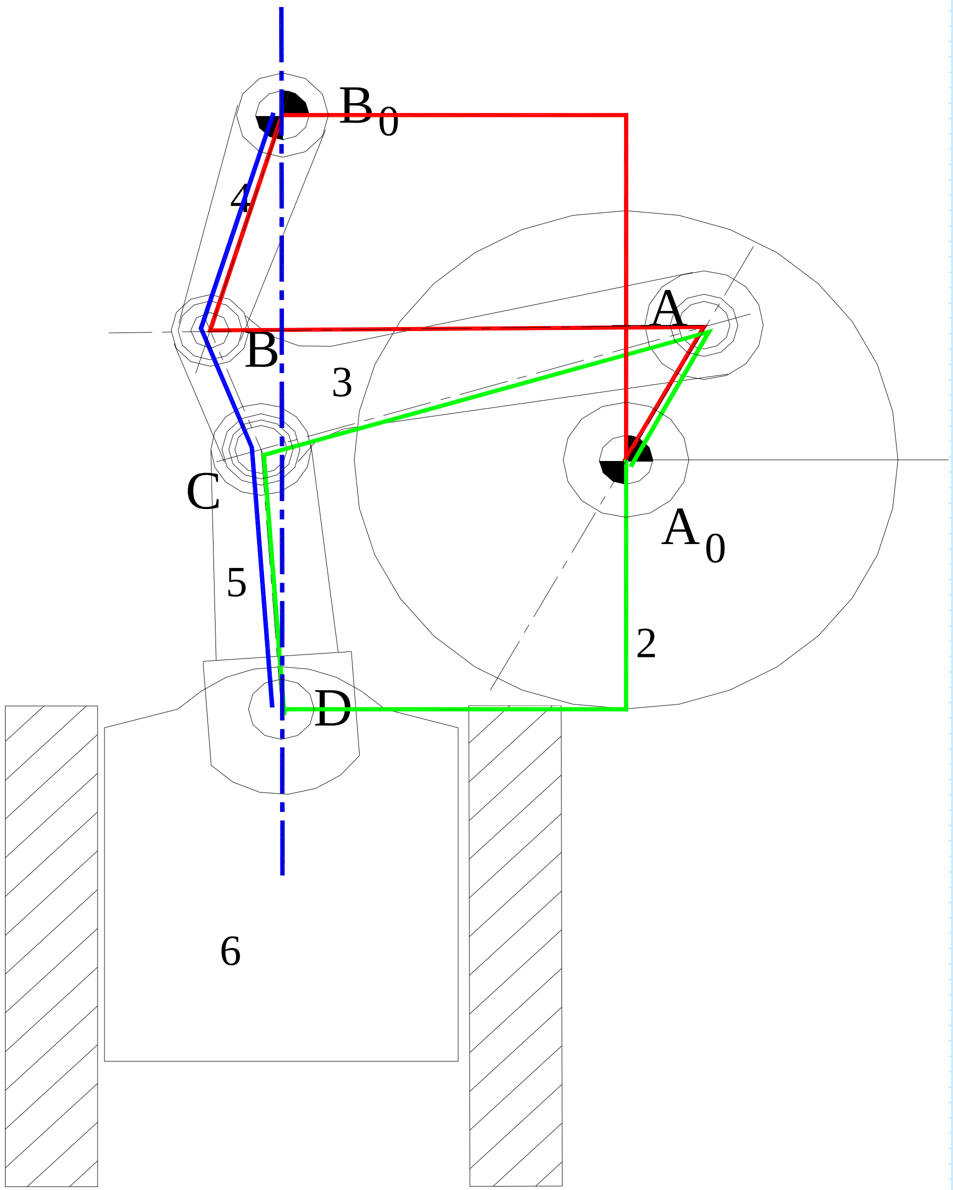


$$\theta_{17} = \cos^{-1}\left[\frac{1}{a_7}\left(c_1 - b_5\cos(\theta_{15}+\alpha_5)\right)\right] + \pi$$

$$s_{18} = b_5\cos(\theta_{15}+\alpha_5) + a_7\sin\theta_{17}$$

Mechanical Press



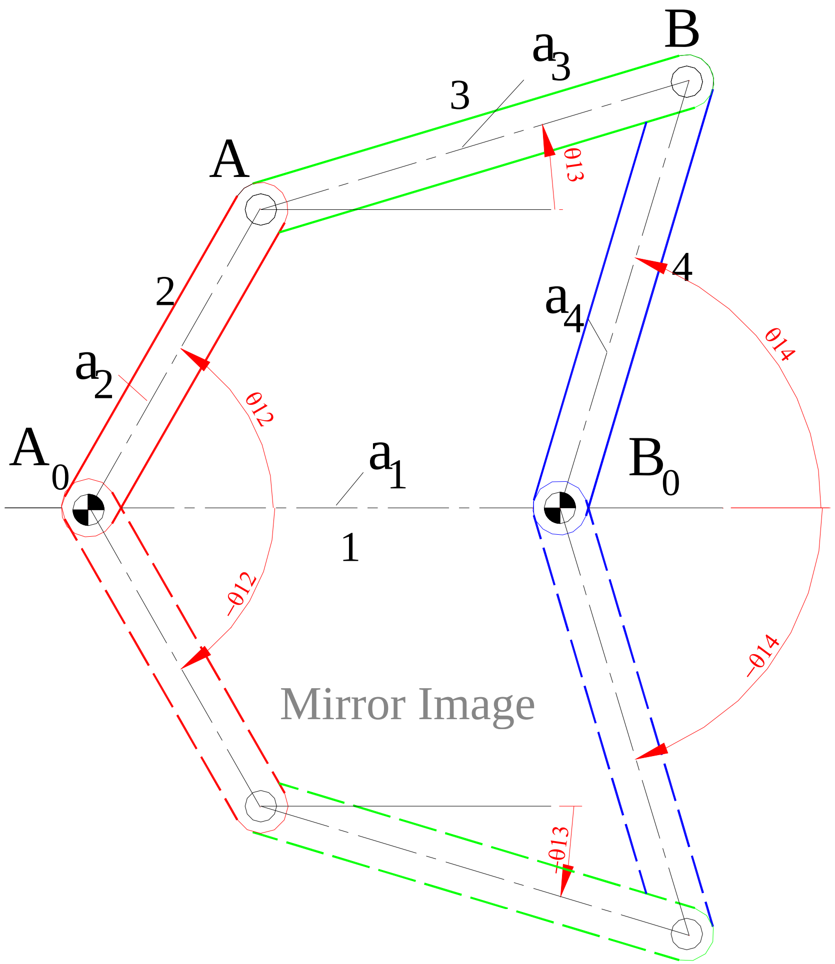


A_0ABB_0 loop: A four-bar

A_0ACD loop

B_0BCD loop

Analytical Closed Form solution of the Loop Equations



Our aim is to obtain θ_{14} as a function of θ_{12}

Loop Closure Equation
for the Four-Bar:

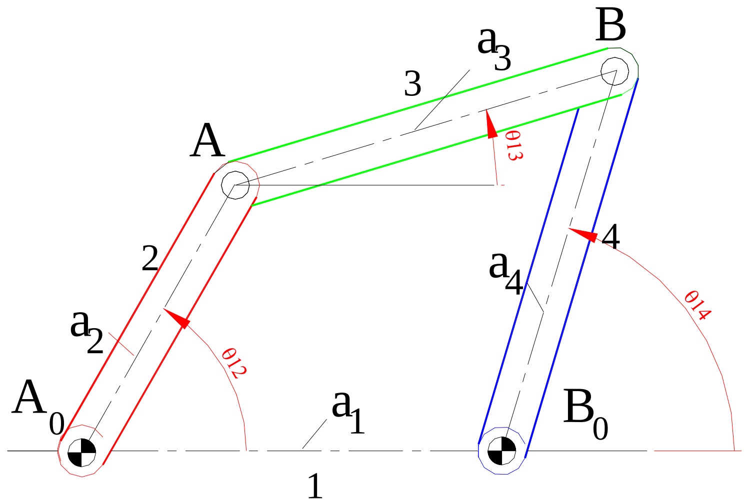
$$a_2 e^{i\theta_{12}} + a_3 e^{i\theta_{13}} = a_1 + a_4 e^{i\theta_{14}}$$

Loop Closure Equation
for the *Mirror Image*:

$$a_2 e^{-i\theta_{12}} + a_3 e^{-i\theta_{13}} = a_1 + a_4 e^{-i\theta_{14}}$$

(This is the complex conjugate of the loop equation)

These two equations yield two complex equations in the complex domain. When we equate the real and imaginary parts separately, the two equations yield two identical scalar equations.



Write the loop Equation and its Complex conjugate in the form:

$$a_3 e^{i\theta_{13}} = a_1 + a_4 e^{i\theta_{14}} - a_2 e^{i\theta_{12}}$$

$$a_3 e^{-i\theta_{13}} = a_1 + a_4 e^{-i\theta_{14}} - a_2 e^{-i\theta_{12}}$$

Multiply the two equations Side by side

$$a_3^2 e^{i(\theta_{13}-\theta_{13})} = (a_1 + a_4 e^{i\theta_{14}} - a_2 e^{i\theta_{12}})(a_1 + a_4 e^{-i\theta_{14}} - a_2 e^{-i\theta_{12}})$$

$$e^{i(\theta-\theta)} = e^{i0} = 1; \qquad \cos\theta = (e^{i\theta} + e^{-i\theta})/2$$

$$K_1 \cos\theta_{14} - K_2 \cos\theta_{12} + K_3 = \cos(\theta_{14} - \theta_{12})$$

Freudenstein's Equation

$$K_1 = \frac{a_1}{a_2} \qquad K_2 = \frac{a_1}{a_4} \qquad K_3 = \frac{(a_1^2 + a_2^2 - a_3^2 + a_4^2)}{2a_4 a_2}$$

$$K_1 \cos\theta_{14} - K_2 \cos\theta_{12} + K_3 = \cos\theta_{14}\cos\theta_{12} + \sin\theta_{14}\sin\theta_{12}$$

$$K_1 \cos\theta_{14} - K_2 \cos\theta_{12} + K_3 = \cos\theta_{14} \cos\theta_{12} + \sin\theta_{14} \sin\theta_{12}$$

$$\sin\theta_{14} = \frac{2\tan(\frac{1}{2}\theta_{14})}{\left[1+\tan^2\left(\frac{1}{2}\theta_{14}\right)\right]} \qquad \cos\theta_{14} = \frac{\left[1-\tan^2\left(\frac{1}{2}\theta_{14}\right)\right]}{\left[1+\tan^2\left(\frac{1}{2}\theta_{14}\right)\right]}$$

$$A\tan^2\left(\frac{\theta_{14}}{2}\right) + B\tan\left(\frac{\theta_{14}}{2}\right) + C = 0$$

$$A = \cos\theta_{12}(1-K_2) + K_3 - K_1$$

$$B = -2\sin\theta_{12}$$

$$C = \cos\theta_{12}(1+K_2) + K_3 + K_1$$

$$\tan\left(\frac{\theta_{14}}{2}\right) = \frac{-B \pm \sqrt{(B^2 - 4AC)}}{2A} \quad \text{or} \quad \theta_{14} = 2\tan^{-1}\left[\frac{-B \pm \sqrt{(B^2 - 4AC)}}{2A}\right]$$

$$K_1 \cos\theta_{14} - K_2 \cos\theta_{12} + K_3 = \cos\theta_{14}\cos\theta_{12} + \sin\theta_{14}\sin\theta_{12}$$

$$(K_1 - \cos\theta_{12})\cos\theta_{14} - \sin\theta_{14}\sin\theta_{12} = K_2\cos\theta_{12} - K_3$$

Let

$$D\cos\phi = K_1 - \cos\theta_{12}$$

$$D\sin\phi = \sin\theta_{12}$$

$$D = \sqrt{(K_1 - \cos\theta_{12})^2 + \sin^2\theta_{12}} = \sqrt{(K_1^2 + 1 - 2K_1\cos\theta_{12})}$$

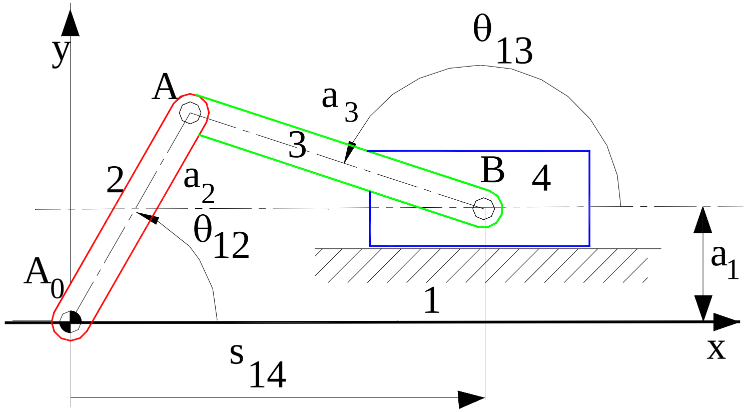
$$\phi = \tan^{-1}\left(\frac{\sin\theta_{12}}{K_1 - \cos\theta_{12}}\right)$$

$$D\cos\phi\cos\theta_{14} - D\sin\phi\sin\theta_{14} = K_2\cos\theta_{12} - K_3$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\cos(\theta_{14} + \phi) = \frac{K_2\cos\theta_{12} - K_3}{D}$$

$$\theta_{14} = -\phi + \cos^{-1}\left(\frac{K_2\cos\theta_{12} - K_3}{D}\right)$$



Loop Closure Eq. and its Complex Conjugate

$$a_2 e^{i\theta_{12}} = s_{14} + i a_1 + a_3 e^{i\theta_{13}}$$

$$a_2 e^{-i\theta_{12}} = s_{14} - i a_1 + a_3 e^{-i\theta_{13}}$$

Eliminate θ_{13}: $a_3 e^{i\theta_{13}} = a_2 e^{i\theta_{12}} - s_{14} - i a_1$

$$a_3 e^{-i\theta_{13}} = a_2 e^{-i\theta_{12}} - s_{14} + i a_1$$

Multiply:

$$a_3^2 = a_2^2 + s_{14}^2 + a_1^2 - s_{14} a_2 (e^{i\theta_{12}} + e^{-i\theta_{12}}) + i a_1 a_2 (e^{i\theta_{12}} - e^{-i\theta_{12}})$$

$\cos\theta = (e^{i\theta} + e^{-i\theta}) / 2$ $\qquad i \sin\theta = (e^{i\theta} - e^{-i\theta}) / 2$

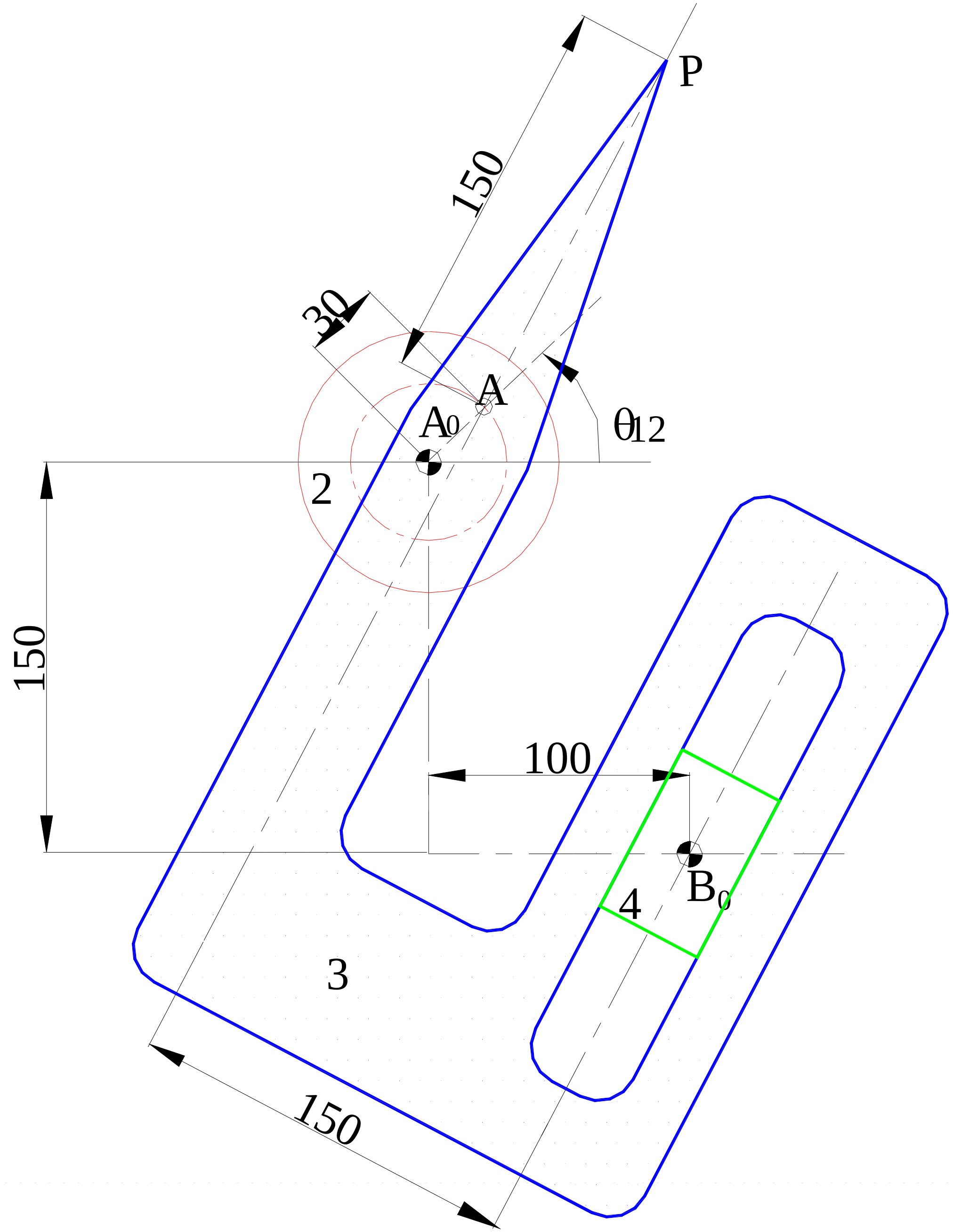
We obtain a quadratic equation in s_{14}:

$$s_{14}^2 - 2 s_{14} a_2 \cos\theta_{12} + (a_1^2 + a_2^2 - a_3^2 - 2 a_1 a_2 \sin\theta_{12}) = 0$$

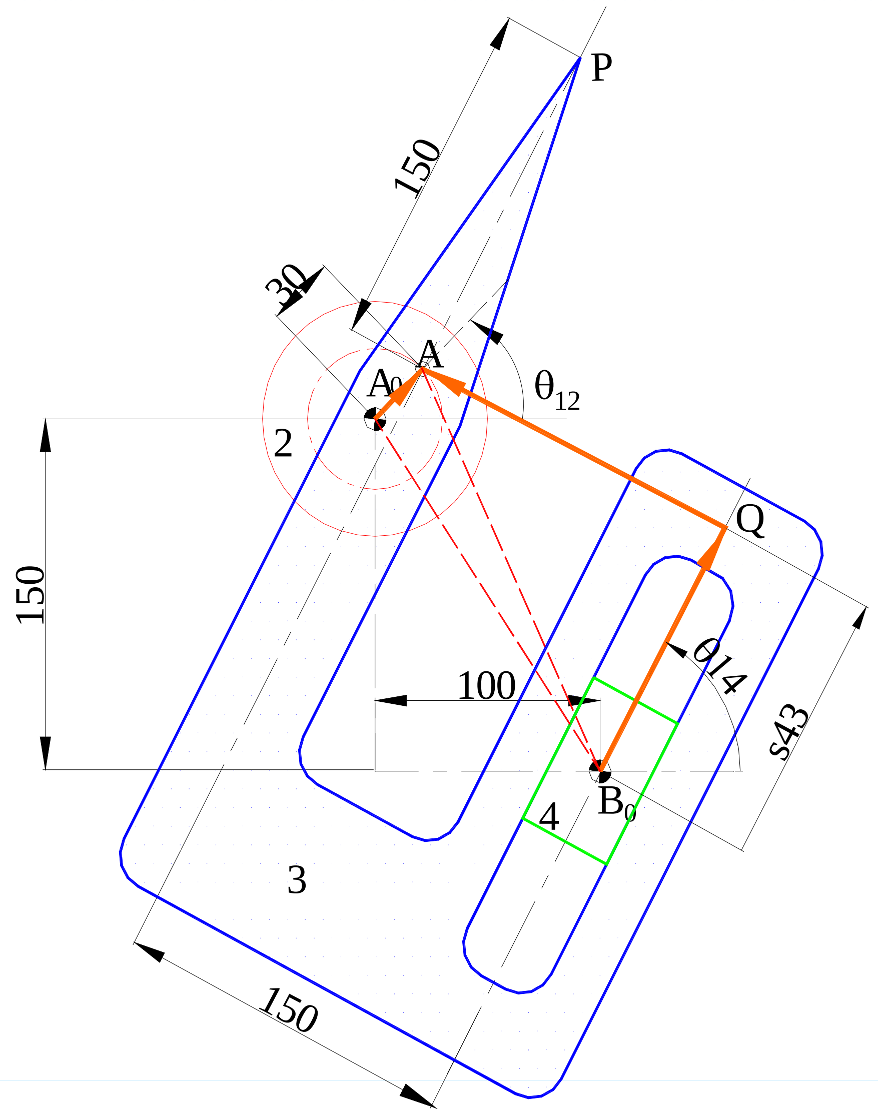
$$s_{14} = a_2 \cos\theta_{12} \pm \sqrt{\left(a_3^2 - a_2^2 \sin^2\theta_{12} - a_1^2 + 2 a_1 a_2 \sin\theta_{12}\right)}$$

"Solving problems is a practical art, like swimming or skiing, or playing piano: you can learn it only by imitation and practice. This book cannot offer you a magic key that opens all the doors and solves all the problems, but it offers you good examples for imitation and many opportunities for practice: if you wish to learn swimming you have to go into the water and if you wish to become a problem solver you have to solve problems "

George Polya, **Mathematical Discovery**



P
150
30
A
A_0
θ_{12}
2
150
100
4
B_0
3
150



$A_0A=A_0B_0+B_0Q+QA$

$A_0A=a_2=30$

$AQ=a_3=150$

$A_0B_0=a_1-ib_1=100-i150$

$$a_2e^{i\theta_{12}} = a_1 - ib_1 + s_{43}e^{i\theta_{14}} + ia_3e^{i\theta_{14}}$$

$$a_2e^{-i\theta_{12}} = a_1 + ib_1 + s_{43}e^{-i\theta_{14}} - ia_3e^{-i\theta_{14}}$$

In StepWise Solution:

1. **Determine A_0B_0**
2. **Determine AB_0 and its angular orientation**
3. **Solve Triangle AB_0Q**

Analytical Solution: Eliminate s_{43}

$$a_2 e^{i\theta_{12}} = a_1 - ib_1 + s_{43} e^{i\theta_{14}} + ia_3 e^{i\theta_{14}} \qquad s_{43} e^{i\theta_{14}} = a_2 e^{i\theta_{12}} - a_1 + ib_1 - ia_3 e^{i\theta_{14}}$$

$$a_2 e^{-i\theta_{12}} = a_1 + ib_1 + s_{43} e^{-i\theta_{14}} - ia_3 e^{-i\theta_{14}} \qquad s_{43} e^{-i\theta_{14}} = a_2 e^{-i\theta_{12}} - a_1 - ib_1 + ia_3 e^{-i\theta_{14}}$$

Take the ratio and cross multiply

$$a_2 e^{i(\theta_{12}-\theta_{14})} - a_1 e^{-i\theta_{14}} + ib_1 e^{-i\theta_{14}} - ia_3 = a_2 e^{-i(\theta_{12}-\theta_{14})} - a_1 e^{i\theta_{14}} - ib_1 e^{i\theta_{14}} + ia_3$$

Group Terms:

$$a_2 (e^{i(\theta_{12}-\theta_{14})} - e^{-i(\theta_{12}-\theta_{14})}) + a_1 (e^{i\theta_{14}} - e^{-i\theta_{14}}) + ib_1 (e^{i\theta_{14}} + e^{-i\theta_{14}}) - 2ia_3 = 0$$

$$2i \sin\theta = (e^{i\theta} - e^{-i\theta})$$

$$a_2 \sin(\theta_{12} - \theta_{14}) + a_1 \sin\theta_{14} + b_1 \cos\theta_{14} - a_3 = 0$$

$$a_2 \sin\theta_{12} \cos\theta_{14} - a_2 \cos\theta_{12} \sin\theta_{14} + a_1 \sin\theta_{14} + b_1 \cos\theta_{14} - a_3 = 0$$

$$\cos\theta_{14} \left[b_1 + a_2 \sin\theta_{12} \right] + \sin\theta_{14} \left[a_1 - a_2 \cos\theta_{12} \right] = a_3$$

Let

$$D\cos\phi = b_1 + a_2\cos\theta_{12}$$

$$D\sin\phi = a_1 - a_2\sin\theta_{12}$$

$$D = \sqrt{(b_1 + a_2\sin\theta_{12})^2 + (a_1 - a_2\cos\theta_{12})^2} = \sqrt{\left(b_1^2 + a_2^2 + a_1^2 + 2b_1a_2\sin\theta_{12} - 2a_1a_2\cos\theta_{12}\right)}$$

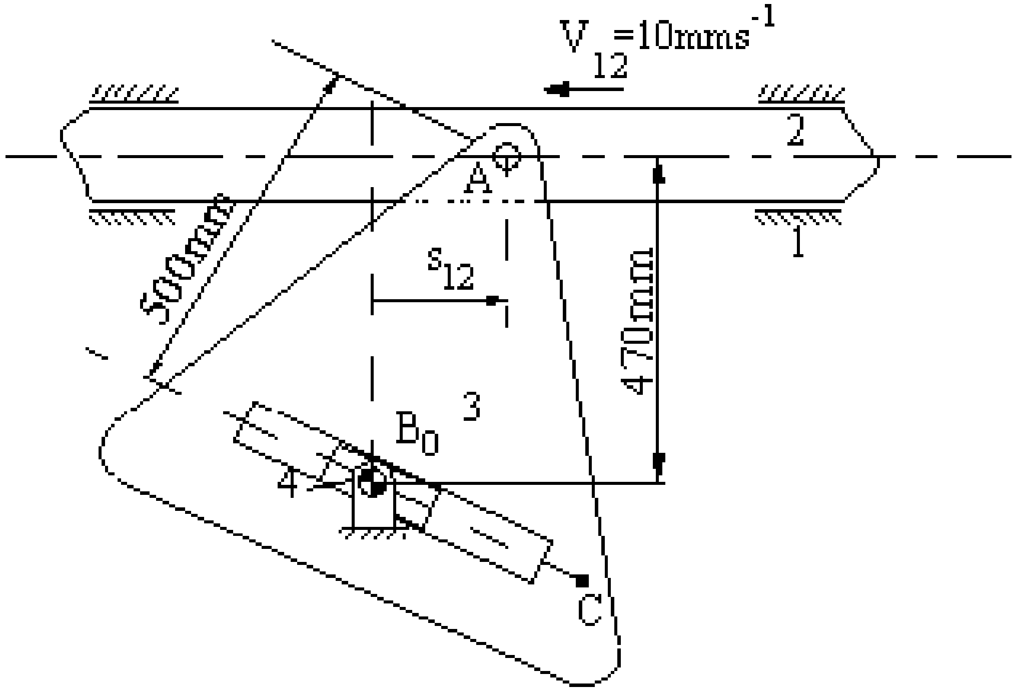
$$\phi = \tan^{-1}\left(\frac{a_1 - a_2\cos\theta_{12}}{b_1 + a_2\sin\theta_{12}}\right)$$

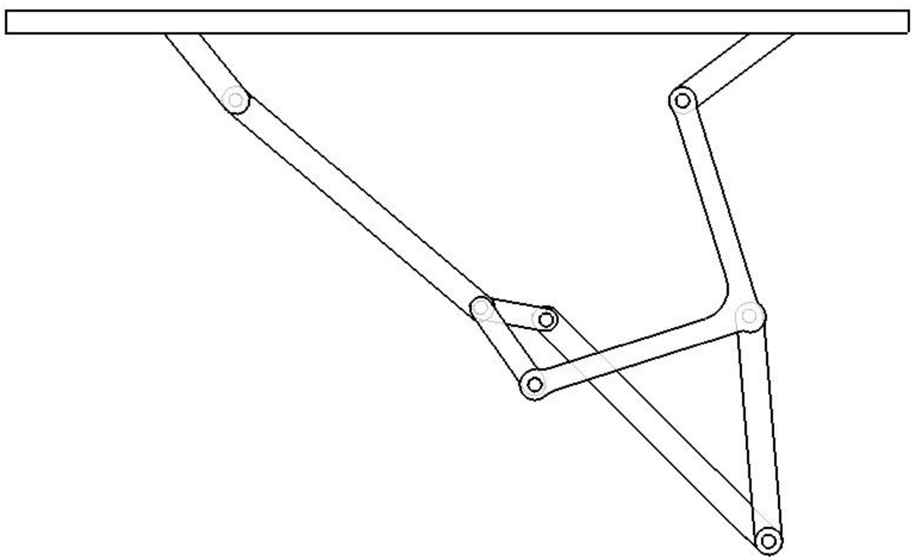
$$D\cos\phi\cos\theta_{14} + D\sin\phi\sin\theta_{14} = a_3$$

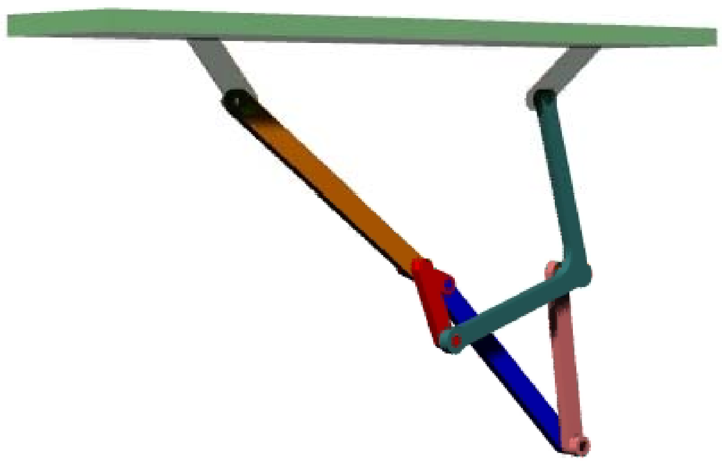
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\cos(\theta_{14} - \phi) = \frac{a_3}{D}$$

$$\theta_{14} = +\phi + \cos^{-1}\left(\frac{a_3}{D}\right)$$


V_{12}=10mms^{-1}
2
1
A
500mm
s_{12}
470mm
B_0
3
4
C



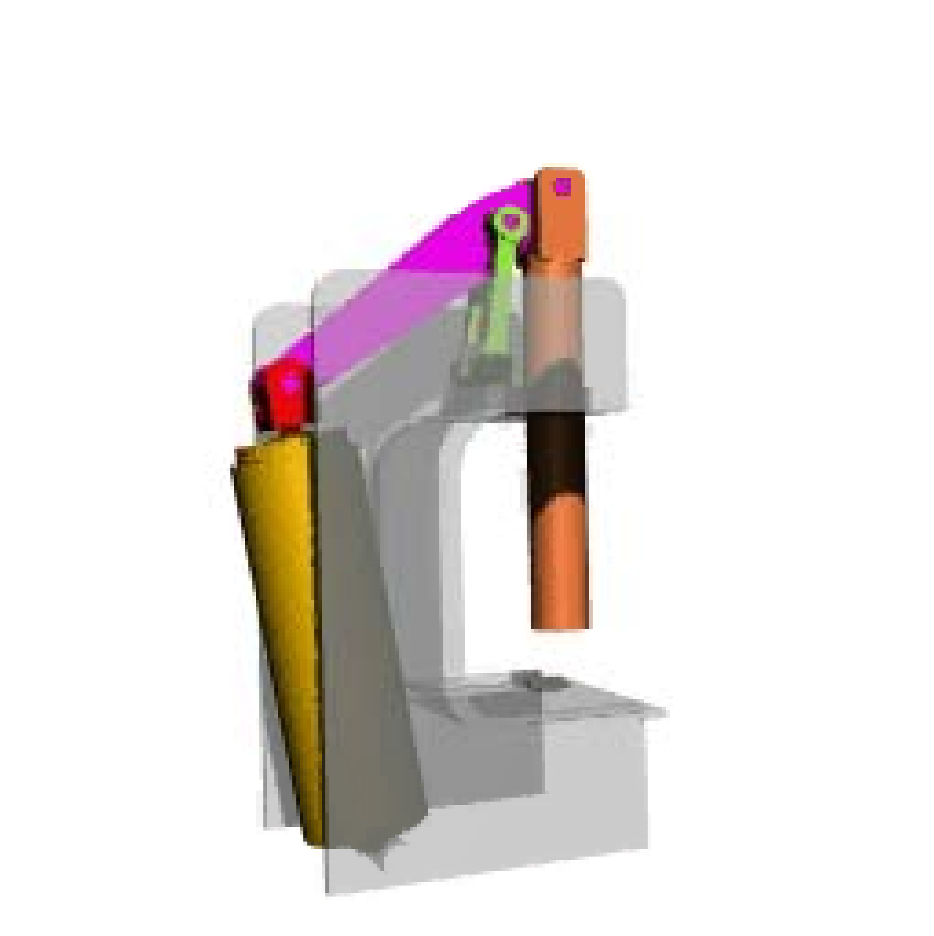


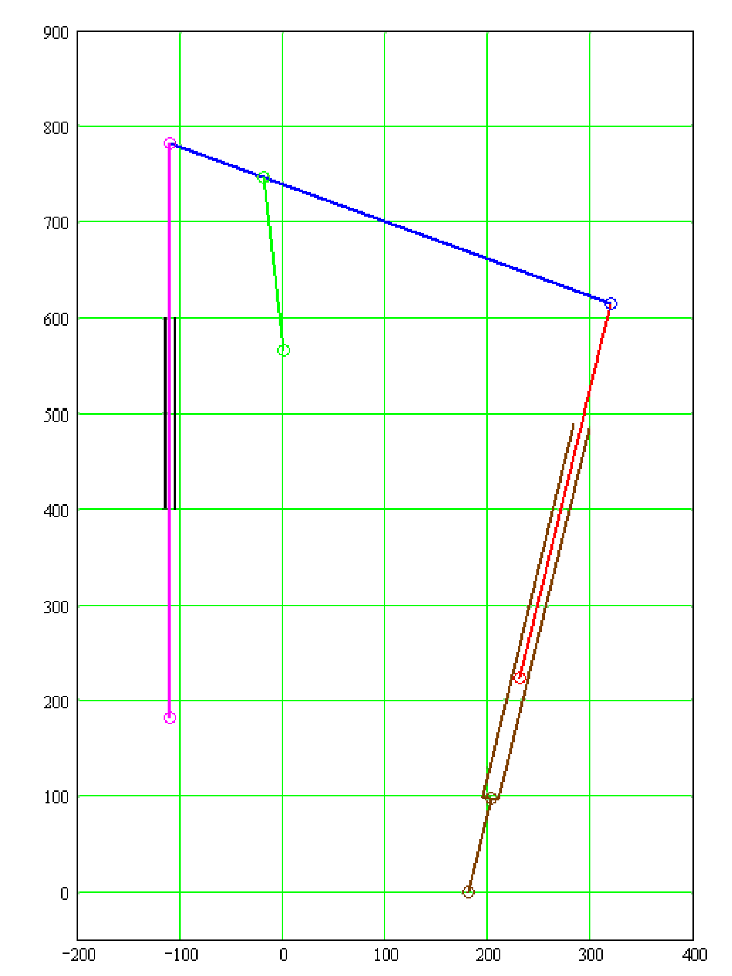
1. **Stepwise method is an exact and efficient method for the analysis of planar mechanisms. It can be used in any computer package with mathematical computation facilities (i.e. MathCad ©, MathLab ©, Excel ©, Eureka ©, etc.) and can be programmed in any computer language (Pascal, C, Basic, etc) or platform (Visual C++, Visual Basic, Delphi, etc), or you can use this method in your calculators as well.**
2. **Graphical method gives a very good insight to a mechanism problem. The Graphical method coincides exactly with the stepwise method. You can use any drafting package (AutoCad©, CadKey ©, ProDesign ©, Ideas ©, SolidWorks ©, etc.) In most of these files you can perform kinematic analysis.**
3. **Analytical solutions are possible for simple single loop mechanisms. They can also be used similar to stepwise solution if there are several loops involved.**
4. **Analytical solutions are sometimes required when performing more complex mechanism analysis or synthesis.**

However

There is a class of planar mechanisms (mechanisms with higher complexity) which cannot be solved analytically or using stepwise procedure.

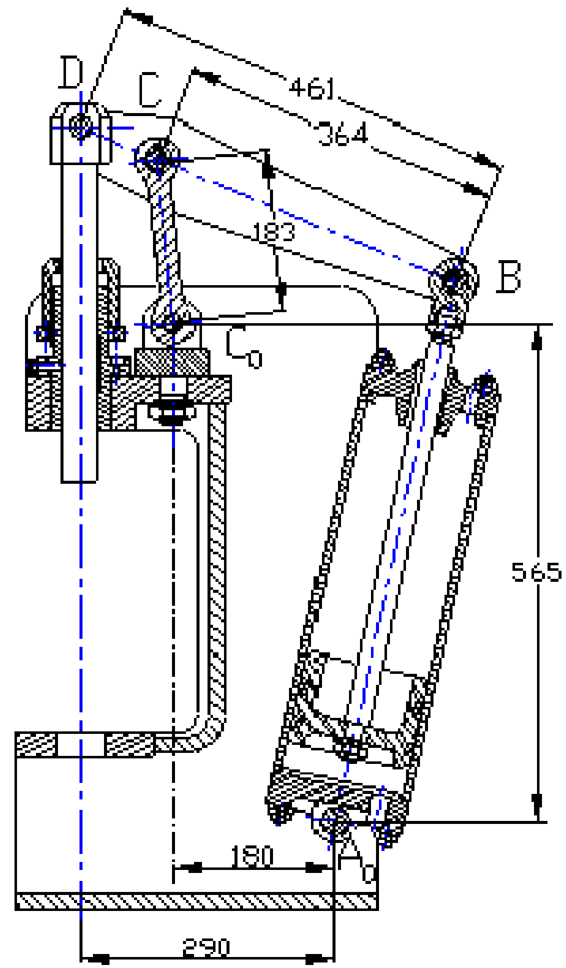
Example

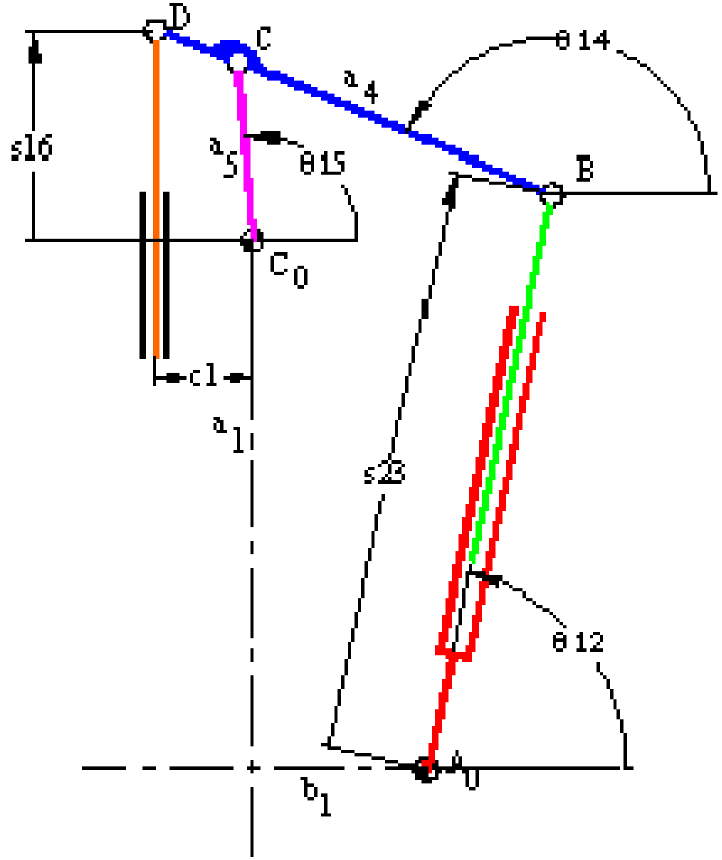




Example

BC=a_4,
CD=c_4,
BD=b_4





Requires simultaneous solution of the two loops!!!

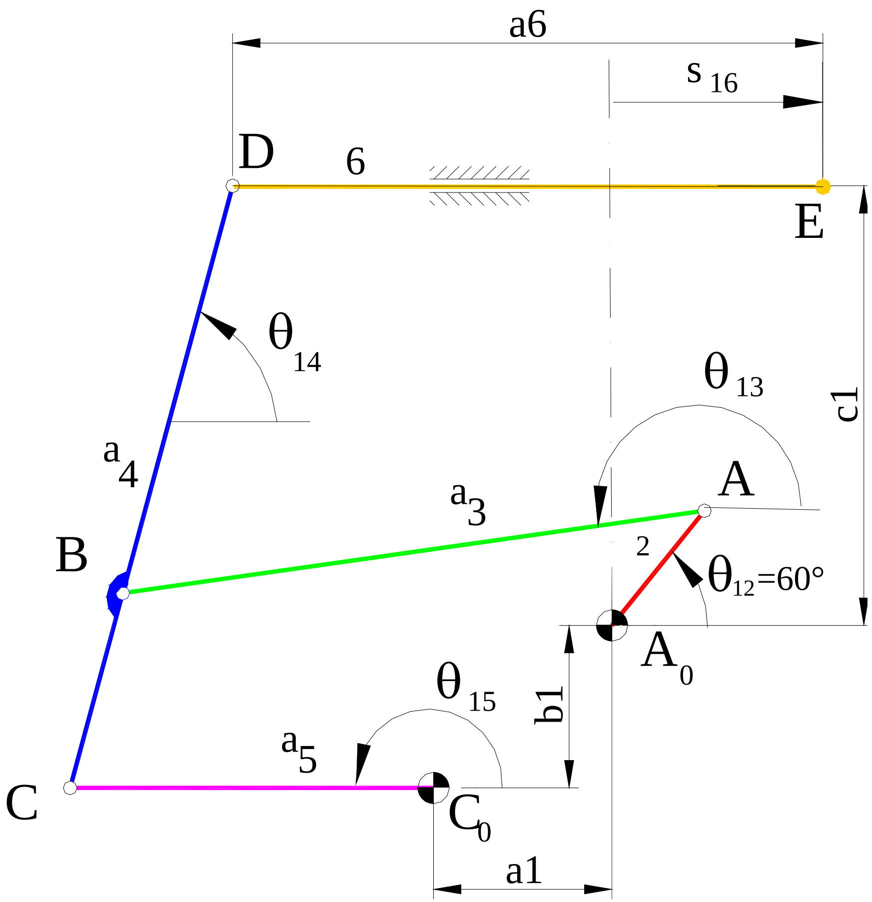
$$s_{23}e^{i\theta_{12}} + a_4 e^{i\theta_{14}} = -b_1 + ia_1 + a_5 e^{i\theta_{15}}$$

$$s_{23}e^{i\theta_{12}} + b_4 e^{i\theta_{14}} = -(b_1 + c_1) + i(a_1 + s_{16})$$

or

$$a_5 e^{i\theta_{15}} + c_4 e^{i\theta_{14}} = -c_1 + is_{16}$$

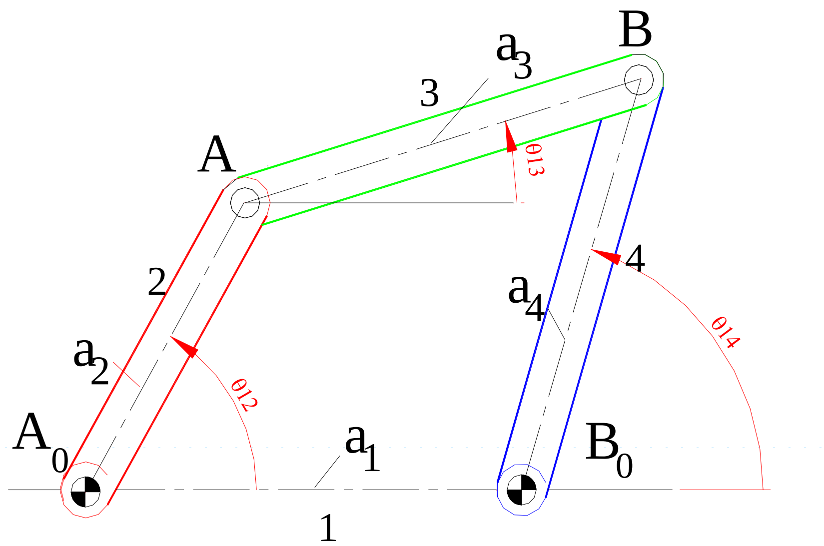
Example



$$a_2 e^{i\theta_{12}} + a_3 e^{i\theta_{13}} = -a_1 - ib_1 + a_5 e^{i\theta_{15}} + b_4 e^{i\theta_{14}}$$

$$a_2 e^{i\theta_{12}} + a_3 e^{i\theta_{13}} + a_4 e^{i\theta_{14}} = ic_1 + (s_{16} - a_6)$$

Iterative Solution of the Loop Closure Equations



$$a_2 e^{i\theta_{12}} + a_3 e^{i\theta_{13}} = a_1 + a_4 e^{i\theta_{14}}$$

$$a_4 \cos(\theta_{14}) - a_3 \cos(\theta_{13}) + a_1 - a_2 \cos(\theta_{12}) = f_1(\theta_{13}, \theta_{14}) = 0$$
$$a_4 \sin(\theta_{14}) - a_3 \sin(\theta_{13})_1 - a_2 \sin(\theta_{12}) = f_2(\theta_{13}, \theta_{14}) = 0$$

Two nonlinear equations in two unknowns. Use a numerical method (such as fixed point or Newton-Raphson method) to solve the set of nonlinear equations

In Excel: Use Solver ADD-in to solve the set of nonlinear equations

In MathCad Use “given”, “Find“ routine.